What are two-point diagnostics telling us in light of H(z) data?

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Outline

- **Background**
- **Method and data**
  - Diagnoses in brief
  - data: H(z) and cosmological parameters for comparison
  - Statistical method and result
- **Conclusion**

Background

The Nobel Prize in Physics 2011

Saul Perlmutter, Brian P. Schmidt, Adam G. Riess

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The Nobel Prize in Physics 2011

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Saul Perlmutter

Prize share: 1/2

Photo: U. Montan

Brian P. Schmidt

Prize share: 1/4

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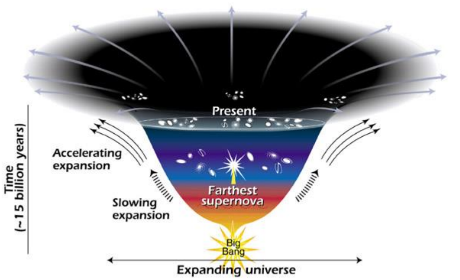
Adam G. Riess

Prize share: 1/4

The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae".*



This diagram reveals changes in the rate of expansion since the universe's birth 15 billion years ago. The more shallow the curve, the faster the rate of expansion. The curve changes noticeably about 7.5 billion years ago, when objects in the universe began flying apart at a faster rate. Astronomers theorize that the faster expansion rate is due to a mysterious, dark force that is pushing galaxies apart.

Why accelerating expansion?

Dark Energy

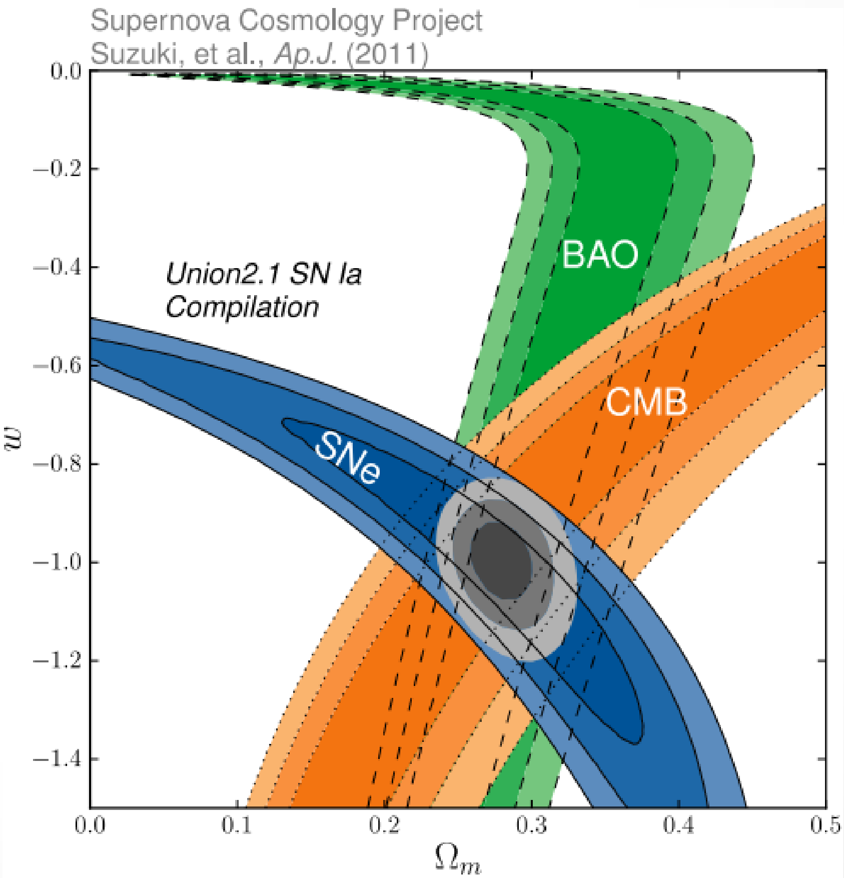
The simplest explanation for dark energy:

Λ

Does ΛCDM model consistent with observational data?

All observational evidence (SNe Ia, CMB, BAO) was concordant with the simplest assumption that there exists non-vanishing cosmological constant Λ.

The ΛCDM model still encounters lots of problems.



THE ASTROPHYSICAL JOURNAL LETTERS, 793:L40 (4pp), 2014 October 1 doi:10.1088/2041-8205/793/2/L40

MODEL-INDEPENDENT EVIDENCE FOR DARK ENERGY EVOLUTION FROM BARYON ACOUSTIC OSCILLATIONS

V. SAHNI[1], A. SHAFIELOO[2,3], AND A. A. STAROBINSKY[4,5]

In the present case $n = 3$, which leads to 3 independent measurements of $Omh^2(z_2; z_1)$, namely

$$
\begin{aligned}
Omh^2(z_1; z_2) &= 0.124 \pm 0.045 \\
Omh^2(z_1; z_3) &= 0.122 \pm 0.01 \\
Omh^2(z_2; z_3) &= 0.122 \pm 0.012 \\
\boxed{\Omega_{m,0} h^2_{Planck} = 0.1426 \pm 0.0025} & \qquad (6)
\end{aligned}
$$

where $z_1 = 0, z_2 = 0.57, z_3 = 2.34$, and the Hubble parameter at these redshifts is $H(z = 0) = 70.6 \pm 3.2$ km/sec/Mpc (Riess et al. 2011; Planck XVI 2013; Delubac et al. 2014), $H(z = 0.57) = 92.4 \pm 4.5$ km/sec/Mpc (Samushia et al. 2013) and $H(z = 2.34) = 222 \pm 7$ km/sec/Mpc (Delubac et al. 2014).

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doi:10.1088/2041-8205/803/2/L22

IS THERE ... UTION?

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Received 2... April 17

Recently, Sahni et al. combined ... AO data with the value of the Hubble constant $H_0 = H(z = 0)$ ... esis by means of an improved version of the Om diagnostic. ... nt between observations and predictions of the Λ cold dark ma... lusion was based only on three measurements of $H(z)$. This moti... By using a comprehensive data set of 29 $H(z)$, we find that discr... $\Omega_{m,0}h^2$ inferred from the Omh^2 diagnostic depends on the way ... ther the weighted mean or the median), the persisting discrepan... DM model may not be the best description of our universe.

Key words: cosmology: observat...

Table 1
Data of the Hubble Parameter $H(z)$ vs. Redshift z, where $H(z)$ and σ_H Are in km s^{-1} Mpc^{-1}

z	$H(z)$	σ_H	Method
0.07	69	19.6	DA
0.1	69	12	DA
0.12	68.6	26.2	DA
0.17	83	8	DA
0.179	75	4	DA
0.199	75	5	DA
0.2	72.9	29.6	DA
0.27	77	14	DA
0.28	88.8	36.6	DA
0.35	82.7	8.4	BAO
0.352	83	14	DA
0.4	95	17	DA
0.44	82.6	7.8	BAO
0.48	97	62	DA
0.57	92.9	7.8	BAO
0.593	104	13	DA
0.6	87.9	6.1	BAO
0.68	92	8	DA
0.73	97.3	7	BAO
0.781	105	12	DA
0.875	125	17	DA
0.88	90	40	DA
0.9	117	23	DA
1.037	154	20	DA
1.3	168	17	DA
1.43	177	18	DA
1.53	140	14	DA
1.75	202	40	DA
2.34	222	7	BAO

Note. These are essentially the data of Farooq & Ratra (2013), with the BAO measurement at the largest redshift $H(z = 2.34)$ taken from Delubac et al. (2015).

New H(z) data from cosmic chronometers:

Table 1. Hubble parameter measurements.

z	H(z) [km/sec/Mpc]
1.363	160 ± 33.6
1.965	186.5 ± 50.4

M., Moresco et al., 2015, Mon. Not. Roy. Astron. Soc., 450, L16 [arXiv:1503.01116]

	M11 models					BC03 models				
z	$H(z)$	σ_{stat}	σ_{syst}	σ_{tot}	% error	$H(z)$	σ_{stat}	σ_{syst}	σ_{tot}	% error
0.3802	89.3	3.2	13.7	14.1	15.8%	83.0	4.3	12.9	13.5	16.3%
0.4004	82.8	2.4	10.3	10.6	12.8%	77.0	2.1	10	10.2	13.2%
0.4247	93.7	2.7	11.4	11.7	12.4%	87.1	2.4	11	11.2	12.9%
0.4497	99.7	3.1	13	13.4	13.4%	92.8	4.5	12.1	12.9	13.9%
0.4783	86.6	2	8.5	8.7	10.1%	80.9	2.1	8.8	9	11.2%
⟨0.4293⟩	91.8	1	5.1	5.3	5.8%	85.7	1	5.1	5.2	6.1%

Table 3. $H(z)$ measurements (in units of [km/Mpc/s]) and their errors. The relative contribution of statistical and systematic errors are reported, as well as the total error (estimated by summing in quadrature σ_{stat} and σ_{syst}). These values have been estimated with M11 and BC03 EPS models respectively. For each model the averaged measurement is also reported. This dataset can be downloaded from http://www.physics-astronomy.unibo.it/en/research/areas/astrophysics/cosmology-with-cosmic-chronometers.

M., Moresco et al., 2016 [arXiv:1601.01701]

We want to answer two main questions in this work:

According to $Om(z_i,z_j)$ and $Omh^2(z_i,z_j)$ diagnostics, does ΛCDM model consistent with the newest H(z) data?

If ΛCDM model in tension with H(z) data, whether the XCDM or CPL models agree better with the H(z) data?

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Diagnostics in Brief

Om(z) diagnostic:

$$\boxed{Om(z) \equiv \frac{\tilde{h}^2(z) - 1}{(1+z)^3 - 1}}$$

Sahni, V., Shafieloo, A. & Starobinsky, A.A., 2008, Phys. R ev. D, 78, 103502 [arXiv:0807.3548]

$$\boxed{\tilde{h}(z) \equiv H(z)/H_0}$$

$$\boxed{H(z)^2{}_{\Lambda CDM} = H_0{}^2[\Omega_{m,0}(1+z)^3 + 1 - \Omega_{m,0}]}$$

Left side: $\boxed{H(z), H_0}$

Right side: $\boxed{Om(z)_{\Lambda CDM} = \Omega_{m,0}}$

- No accurate direct measurements.
- Reconstruct from distance measurements.

- Hard to measure directly.
- Indirectly, exists debate.

Two-point $Om(z_1, z_2)$ diagnostic:

$$Om(z_1, z_2) \equiv Om(z_1) - Om(z_2)$$

$$= \boxed{\frac{\tilde{h}^2(z_1) - 1}{(1+z_1)^3 - 1} - \frac{\tilde{h}^2(z_2) - 1}{(1+z_2)^3 - 1}}$$

Shafielo o, A., Sahni, V. & Starobinsky, A.A., 2012, Phys. Rev. D, 86, 1 03527 [arXiv:1205.2870]

Left side: $\boxed{H(z), H_0}$

Right side: $\boxed{\mathrm{Om}(z_i, z_j)_{\Lambda CDM} = 0}$

Compared to the original Om(z) diagnostic:

- Without any need of knowing matter density parameter.
- A sample of n measurements offers us n(n−1)/2 different values of two point diagnostics.

Two-point $Omh^2(z_1, z_2)$ diagnostic:

$$Omh^2(z_i, z_j) = \boxed{\frac{h^2(z_i) - h^2(z_j)}{(1+z_i)^3 - (1+z_j)^3}}$$

Sahni, V., Shafielo o, A. & Starobinsky, A.A., 2014, Astrophys. J., 793, L40 [arXiv:1406.2209]

$\boxed{h(z) \equiv H(z)/100\ km\ s^{-1}\ Mpc^{-1}}$

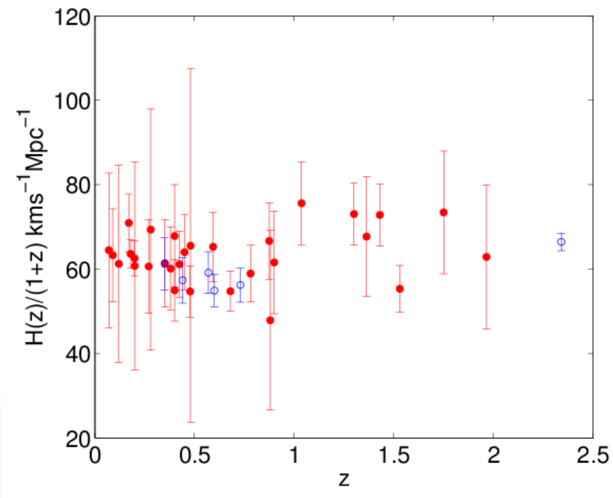
Left side: $\boxed{H(z)}$

Right side:

$$\boxed{\mathrm{Omh}^2(z_i, z_j)_{\Lambda CDM} = \Omega_{m,0} h^2}$$

data (Hubble parameter)

36 H(z)
- 30 Differential Ages(DA)
- 6 Baryon Acoustic Oscillation(BAO)



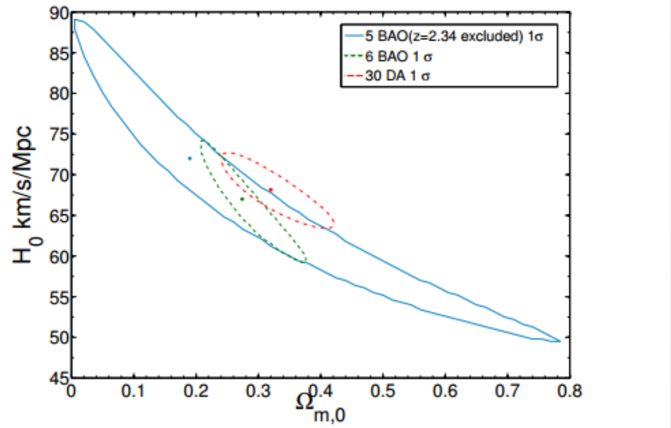
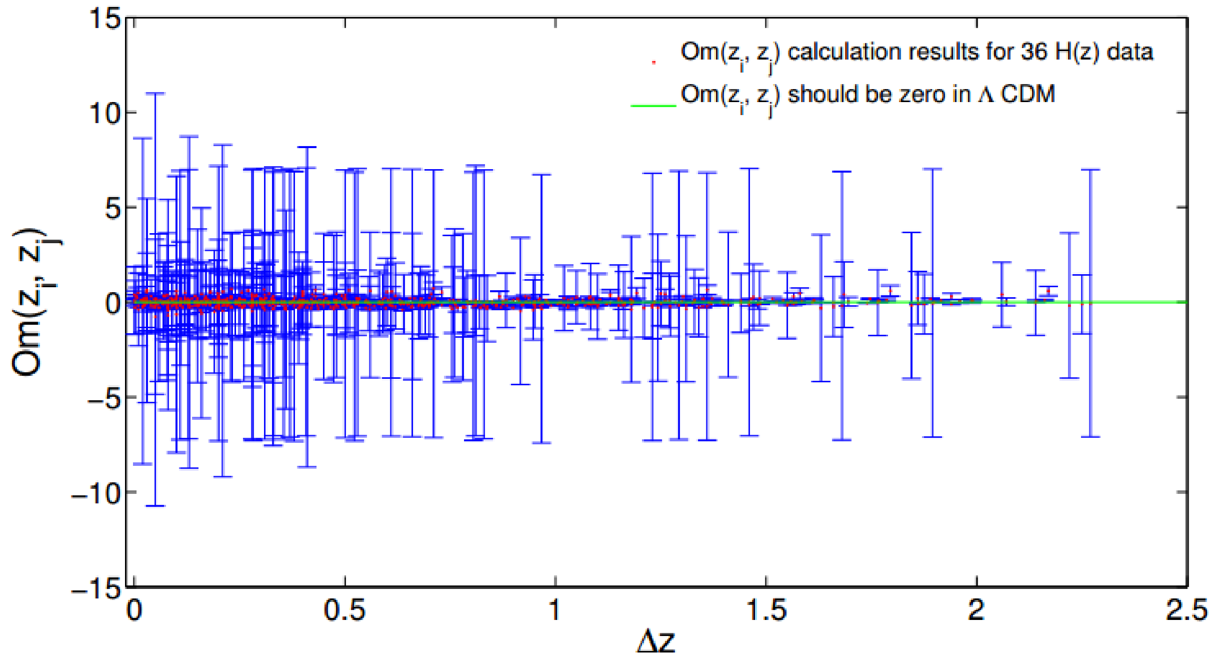


Fig. 1.— Comparison of constraints on the ΛCDM model parameters from $N = 30$ DA data (dash-dot red), $N = 6$ BAO data (dash green) and $N = 5$ BAO with $z = 2.34$ measurement excluded (solid blue). 68% confidence regions are shown with crosses denoting central fits.

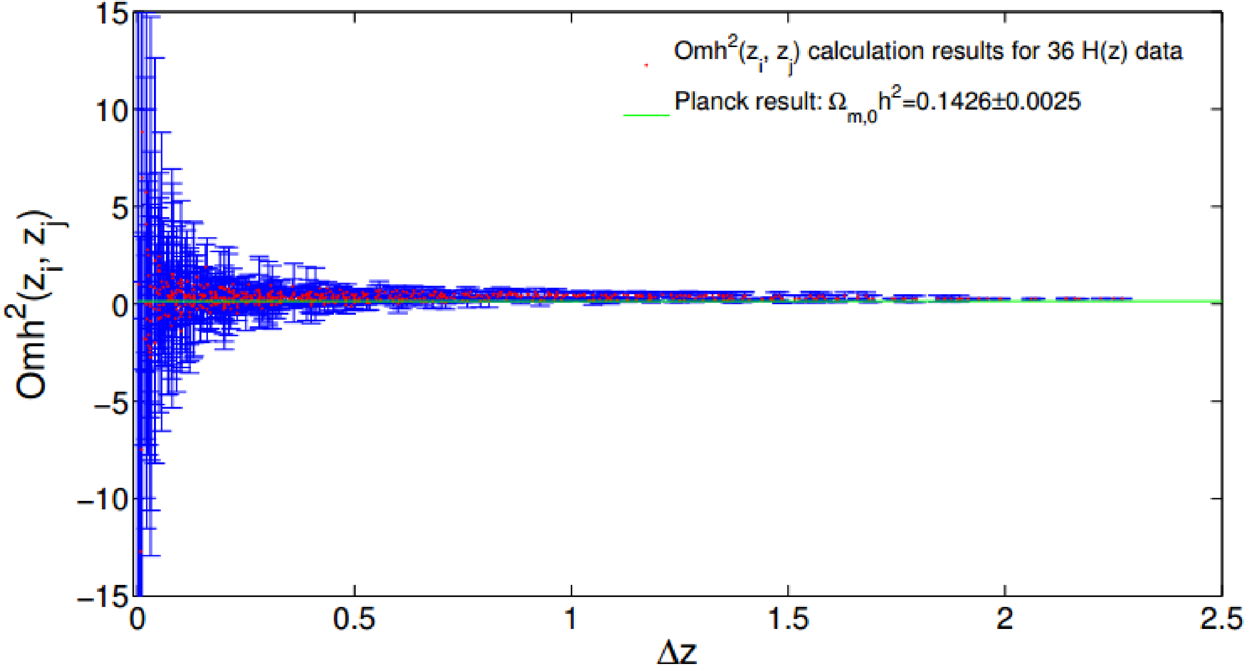
$$Om(z_1, z_2) \equiv Om(z_1) - Om(z_2) = \frac{\tilde{h}^2(z_1) - 1}{(1+z_1)^3 - 1} - \frac{\tilde{h}^2(z_2) - 1}{(1+z_2)^3 - 1}$$

$$\sigma^2_{Om,ij} = \frac{4\tilde{h}^2(z_i)\sigma^2_{\tilde{h}(z_i)}}{((1+z_i)^3 - 1)^2} + \frac{4\tilde{h}^2(z_j)\sigma^2_{\tilde{h}(z_j)}}{((1+z_j)^3 - 1)^2}$$



$$Omh^2(z_i, z_j) = \frac{h^2(z_i) - h^2(z_j)}{(1+z_i)^3 - (1+z_j)^3}$$

$$\sigma^2_{Omh^2,ij} = \frac{4\left(h^2(z_i)\sigma^2_{h(z_i)} + h^2(z_j)\sigma^2_{h(z_j)}\right)}{\left((1+z_i)^3 - (1+z_j)^3\right)^2}$$



Statistical Method

The most popular way of summarizing measurements: weighted mean

For $Omh^2(z_1, z_2)$ diagnostic:

weighted mean:

$$Omh^2_{(w.m.)} = \frac{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} Omh^2(z_i, z_j)/\sigma^2_{Omh^2,ij}}{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} 1/\sigma^2_{Omh^2,ij}}$$

Variance:

$$\sigma^2_{Omh^2_{(w.m.)}} = \left(\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} 1/\sigma^2_{Omh^2,ij} \right)^{-1}$$

This well known and often used approach(weighted mean) relies on several strong assumptions:

- **statistical independence of the data**
- **no systematic effects**
- **Gaussian distribution of the error**

For $Om(z_i, z_j)$ diagnostic is similar.

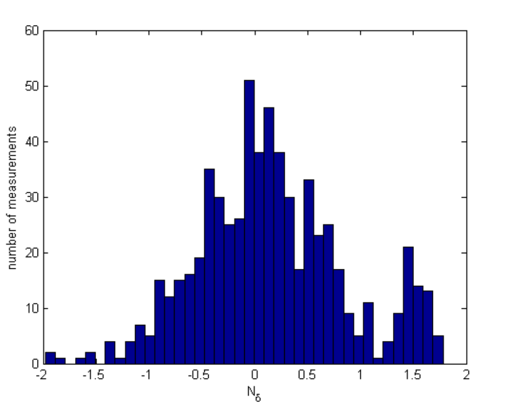
Statistical Method

Therefore, following Chen G. et al. (2003) and Crandall et al. (2014), we have drawn histograms of distribution of our measurements as a function of the number of standard deviations N_σ away from central estimates

$$N_{\sigma,k} = \frac{Omh^2(z_i, z_j) - Omh^2{}_{(w.m.)}}{\sigma_{Omh^2,ij}}$$

For the whole sample, weight mean method, the percentage of the distribution falling within ±1σ, i.e. $|N_\sigma|<1$ is: **83.49%**

For Gaussian distribution,it should be: **68.3%**



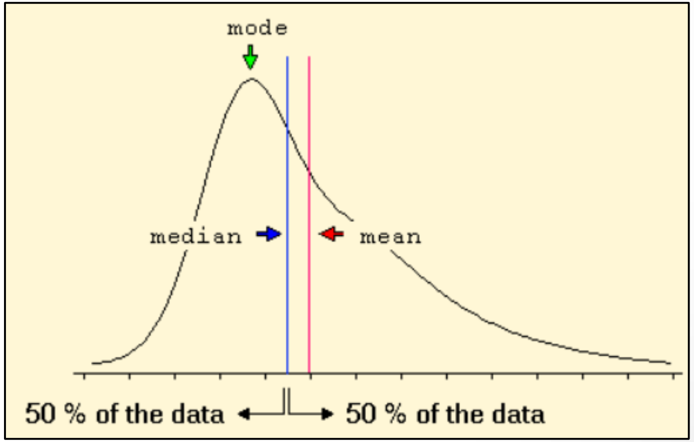
Number of measurements .VS. N_σ

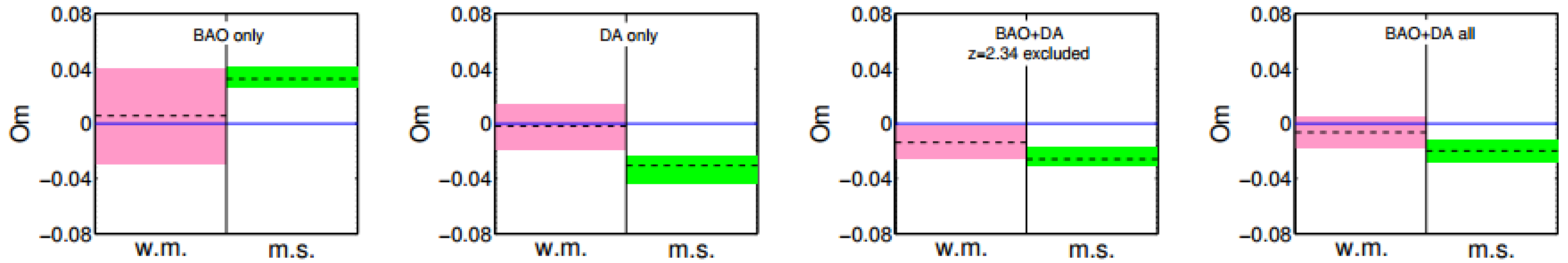
Statistical Method

Another, much more robust approach: calculate the median

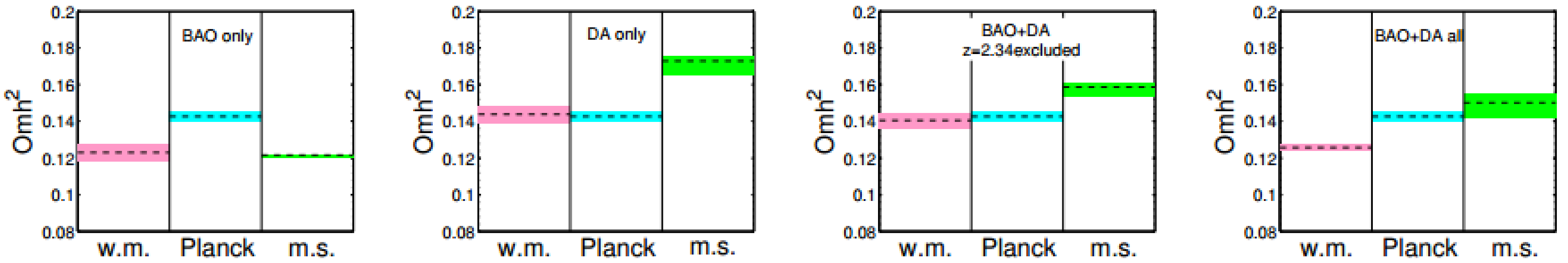
The probability that n observations out of the total number of N is higher than the median follows the binomial distribution:

$$P = 2^{-N} \frac{N!}{n!(N-n)!}$$





The $Om(z_i, z_j)$ two point diagnostic



The $Omh^2(z_i, z_j)$ two point diagnostic

If some other parametrization of dark energy can perform better?

The simplest extensions of the ΛCDM: **wCDM and CPL**

How to do the comparison?

Left side Right side

We calculated the residuals: $R_{\mathrm{Omh}^2}(z_i, z_j) = Omh^2(z_i, z_j) - Omh^2(z_i, z_j)_{th}$

Residuals weighted mean:

$$R_{(w.m.)} = \frac{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} R(z_i, z_j)/\sigma^2_{R,ij}}{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} 1/\sigma^2_{R,ij}}$$

Variance:

$$\sigma^2_{R_{(w.m.)}} = \left(\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} 1/\sigma^2_{R,ij} \right)^{-1}$$

Statistical Method

Left side

Right side

For $Omh^2(z_i, z_j)$ diagnostic:

$$\boxed{Omh^2(z_i,z_j) = \frac{h^2(z_i)-h^2(z_j)}{(1+z_i)^3-(1+z_j)^3}} \Leftrightarrow$$

ΛCDM: $\Omega_{m,0}h^2$

wCDM: $\Omega_{m,0}h^2 + (1-\Omega_{m,0})h^2[\frac{(1+z_i)^{3(1+w)} - (1+z_j)^{3(1+w)}}{(1+z_i)^3 - (1+z_j)^3}]$

CPL:

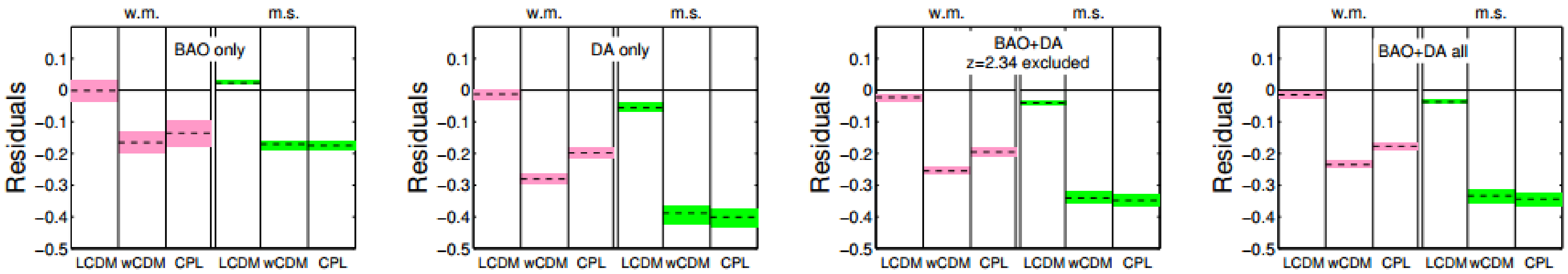
$$\Omega_{m,0}h^2 + (1-\Omega_{m,0})h^2[\frac{(1+z_i)^{3(1+w_0+w_a)} e^{\frac{-3w_a z_i}{1+z_i}} - (1+z_j)^{3(1+w_0+w_a)} e^{\frac{-3w_a z_i}{1+z_i}}}{(1+z_i)^3 - (1+z_j)^3}]$$

Left side: use observational H(z) data from DA and BAO techniques.

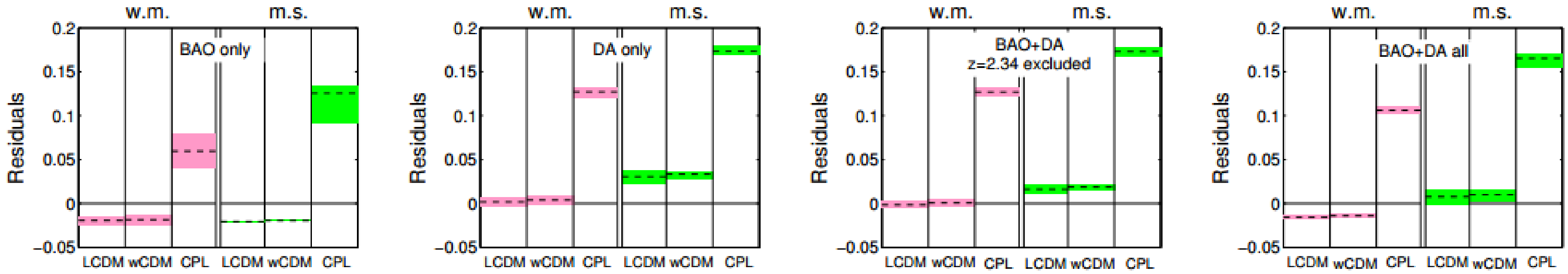
Right side use JLA constraint results:

Table 2: The best-fitted values of parameters for three dark energy models obtained from joint analysis of Planck+WP+BAO+JLA data(Betoule et al. 2014).

	$\Omega_{m,0}$	H_0	w	w_0	w_a
ΛCDM	0.305 ± 0.010	68.34 ± 1.03	□	□	□
$wCDM$	0.303 ± 0.012	68.50 ± 1.27	-1.027 ± 0.055	□	□
CPL	0.304 ± 0.012	68.59 ± 1.27	□	-0.957 ± 0.124	-0.336 ± 0.552



Residuals of $Om(z_i,z_j)$



Residuals of $Omh^2(z_i,z_j)$

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Conclusion and discussion

- 36 H(z) data were used: 30 from DA and 6 from BAO.

  --BAO and DA techniques are prone to different systematic uncertainties.

  --Big leverage of the $z = 2.34$ data point.

- $Om(z_i,z_j)$ and $Omh^2(z_i,z_j)$ diagnostics were discussed to test ΛCDM model.

  --Both two-point diagnostics have non-Gaussian distributions.

  --The median statistic results support the claim that H(z) data seem to be in conflict with the ΛCDM model.

- If other cosmological models, alternative to the ΛCDM perform better?

  --we considered wCDM and CPL models.

  --calculating the "observed - expected" residuals.

  -- ΛCDM is still in better agreement with the data than wCDM or CPL.

- We should pay more attention to the H(z) data from different observation.

Welcome to China and visit our research group!