Stability Properties of Magnetized Spine-Sheath Relativistic Jets

Philip Hardee[1] (UA), Yosuke Mizuno[2] (NSSTC/MSFC) & Ken-Ichi Nishikawa[3] (NSSTC/UAH)

Introduction: Relativistic jets in AGN and microquasars are likely accelerated and collimated by magnetic fields anchored in an accretion disk and/or threading the ergosphere around a rotating black hole. Recent GRMHD numerical results (Mizuno et al.) suggest that a jet spine driven by the magnetic fields threading the ergosphere may be surrounded by a broad jet sheath driven by the magnetic fields anchored in the accretion disk. This configuration might additionally be surrounded by a less highly collimated accretion disk wind from the hot corona. Results from other jet formation simulations suggest that the jet speed is related to the Alfvén wave speed in the acceleration and collimation region implying Alfvén wave speeds near to light speed.

Jet formation simulation results suggest that the theoretical analysis of stability properties and resulting jet structures requires keeping the displacement current in the RMHD equations to allow for strong magnetic fields and Alfvén wave speeds near light speed. Additionally, the recent GRMHD simulation results and existing observational results suggest that jets can have a spine-sheath structure. Thus, a theoretical investigation should allow for flow in a sheath surrounding the jet spine.

In this poster we report on basic theoretical results using the linearized RMHD equations including the displacement current and allowing for flow in a sheath around a faster flowing jet spine. We also report on RMHD numerical simulation results containing an axial magnetic field in both jet spine and sheath. This most stable magnetic configuration is absolutely stable to current driven **(CD)** modes but can be unstable to the surface driven Kelvin-Helmholtz (**KH**) modes.

Possibly the most important basic result is that destructive **KH** modes can be stabilized even when the jet Lorentz factor significantly exceeds the Alfvén Lorentz factor. Even in the absence of stabilization, spatial growth of destructive **KH** modes can be considerably reduced by the presence of marginally relativistic sheath flow (~ 0.5 c) around a relativistic jet spine (> 0.9 c).

This research supported by the National Science Foundation through awards to **(1)** The University of Alabama (UA) in Tuscaloosa and **(3)** the University of Alabama at Huntsville (UAH), and by the NASA Postdoctoral Program through an award to **(2)** the National Space Science & Technology Center (NSSTC) and Marshall Space Flight Center (MSFC) in Huntsville.

GRMHD Simulations of Jet Formation: Spine-Sheath Structure

Key Questions of Jet Formation

- Acceleration to large Lorentz factors
- Collimation & Transverse Structure
- Variability & CD/KH induced structure

New GRMHD code RAISHIN

(Mizuno, Nishikawa et al.)

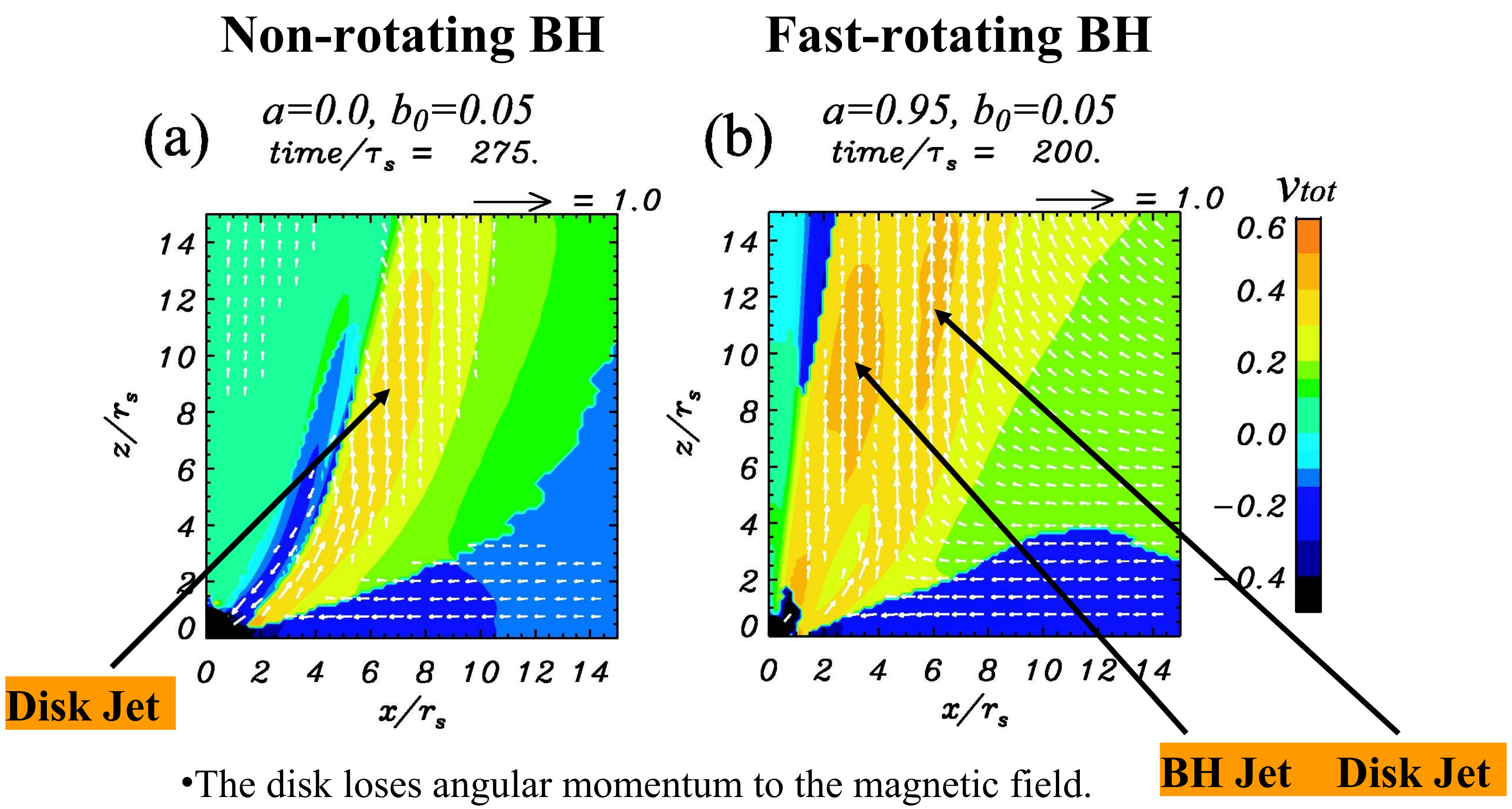
- Multi-dimensional (1D, 2D, 3D)
- Special & General relativity
- Various coordinate systems
- Various boundary conditions
- Divergence free magnetic fields
- Large Lorentz factors

GRMHD Simulation Initial Conditions

- Geometrically thin accretion disk ($\rho_{disk} / \rho_{corona} = 100$)
- Free falling background corona (Bondi Solution)
- Global vertical magnetic field lines (Wald solution)

GRMHD Simulation Results

Simulations were performed for a geometrically thin accretion disk near both non-rotating and rapidly rotating black holes. Similar to previous results (Koide et al. 2000, Nishikawa et al. 2005a) we find magnetically driven jets. It appears that a rapidly rotating black hole creates an inner, faster, and more collimated outflow within a broader jet outflow driven by the accretion disk.



- The disk loses angular momentum to the magnetic field.
- A centrifugal barrier leads to shock formation at r ~ $2R_s$
- J x B and gas pressure forces form the jet(s)
- A hollow jet is formed from twisted magnetic fields anchored in the disk.
- An inner jet is formed from magnetic fields twisted in the BH ergosphere.

Stability Properties of a 3D RMHD Relativistic Jet Spine & Sheath

Dispersion Relation:

$$\frac{\beta_j}{\chi_j}\frac{J_n'(\beta_j R)}{J_n(\beta_j R)} = \frac{\beta_e}{\chi_e}\frac{H_n^{(1)'}(\beta_e R)}{H_n^{(1)}(\beta_e R)}$$

J_n and $H_n^{(1)}$ are Bessel and Hankel functions and primes denote derivatives

$$\chi_j \equiv \gamma_j^2\gamma_{Aj}^2 W_j\left(\varpi_j^2 - \kappa_j^2 v_{Aj}^2\right) \qquad \chi_e \equiv \gamma_e^2\gamma_{Ae}^2 W_e\left(\varpi_e^2 - \kappa_e^2 v_{Ae}^2\right)$$

$$\beta_j^2 \equiv \left[\frac{\gamma_j^2\gamma_{Aj}^2\left(\varpi_j^2 - \kappa_j^2 a_j^2\right)\left(\varpi_j^2 - \kappa_j^2 v_{Aj}^2\right)}{\left(a_j^2 + \gamma_{Aj}^2 v_{Aj}^2\right)\varpi_j^2 - \gamma_{Aj}^2\kappa_j^2 v_{Aj}^2 a_j^2}\right] \qquad \beta_e^2 \equiv \left[\frac{\gamma_e^2\gamma_{Ae}^2\left(\varpi_{ex}^2 - \kappa_e^2 a_e^2\right)\left(\varpi_e^2 - \kappa_e^2 v_{Ae}^2\right)}{\left(a_e^2 + \gamma_{Ae}^2 v_{Ae}^2\right)\varpi_e^2 - \gamma_{Ae}^2\kappa_e^2 v_{Ae}^2 a_e^2}\right]$$

$$\varpi_{j,e}^2 \equiv \left(\omega - k u_{j,e}\right)^2 \qquad \kappa_{j,e}^2 \equiv \left(k - \omega u_{j,e}/c^2\right)^2$$

$$\gamma_{j,e} \equiv \left(1 - u_{j,e}^2/c^2\right)^{-1/2} \qquad \gamma_{sj,e} \equiv \left(1 - a_{j,e}^2/c^2\right)^{-1/2} \qquad \gamma_{Aj,e} \equiv \left(1 - v_{Aj,e}^2/c^2\right)^{-1/2}$$

$a = sound\ speed \qquad v_A = Alfven\ wave\ speed$

Resonance (ω*) :

$$\frac{u_j - u_e}{1 - u_j u_e/c^2} > \frac{v_{wj} + v_{we}}{1 + v_{wj}v_{we}/c^2}$$

$v_{wj} \equiv (a_j, v_{Aj})$ and $v_{we} \equiv (a_e, v_{Ae})$ in (fluid,magnetic) limits

$$v_w \approx v_w^* \equiv \frac{\gamma_j(\gamma_{we}v_{we})u_j + \gamma_e(\gamma_{wj}v_{wj})u_e}{\gamma_j(\gamma_{we}v_{we}) + \gamma_e(\gamma_{wj}v_{wj})}$$

$\gamma_{wj,e} \equiv (1 - v_{wj,e}^2/c^2)^{-1/2}$ for $v_{wj} \equiv (a_j, v_{Aj})$ and $v_{we} \equiv (a_e, v_{Ae})$ in (fluid,magnetic) limits

$$\omega R/v_{we} \approx \omega_{nm}^* R/v_{we} \equiv \frac{(2n+1)\pi/4 + m\pi}{\left[(1 - u_e/v_w^*)^2 - (v_{we}/v_w^* - u_e v_{we}/c^2)\right]^{1/2}}$$

$$\lambda_{nm}^* \equiv \frac{2\pi}{(2n+1)\pi/4 + m\pi}\left(\frac{\gamma_e}{v_{we}}\right)\left[\left(v_w^* - u_e\right)^2 - \left(v_{we} - (v_{we}u_e/c^2)v_w^*\right)^2\right]^{1/2} R$$

Surface Modes @ ω << ω*

$$\frac{\omega}{k} = \frac{[\eta u_j + u_e] \pm i\eta^{1/2}\left[(u_j - u_e)^2 - V_{As}^2/\gamma_j^2\gamma_e^2\right]^{1/2}}{(1 + V_{Ae}^2/\gamma_e^2 c^2) + \eta(1 + V_{Aj}^2/\gamma_j^2 c^2)}$$

$$\eta \equiv \gamma_j^2 W_j/\gamma_e^2 W_e$$

$$V_{As}^2 \equiv \left(\gamma_{Aj}^2 W_j + \gamma_{Ae}^2 W_e\right)\frac{B_j^2 + B_e^2}{4\pi W_j W_e}$$

$$W_{j,e} \equiv \rho_{j,e} + [\Gamma/(\Gamma - 1)]P_{j,e}/c^2$$

Body Mode Condition:

$$\left[\frac{a_j^2 u_j^2 + \gamma_{Aj}^2 v_{Aj}^2(u_j^2 - a_j^2)}{\gamma_j^2\gamma_{Aj}^2(u_j^2 - a_j^2)(u_j^2 - v_{Aj}^2)}\right] > 0$$

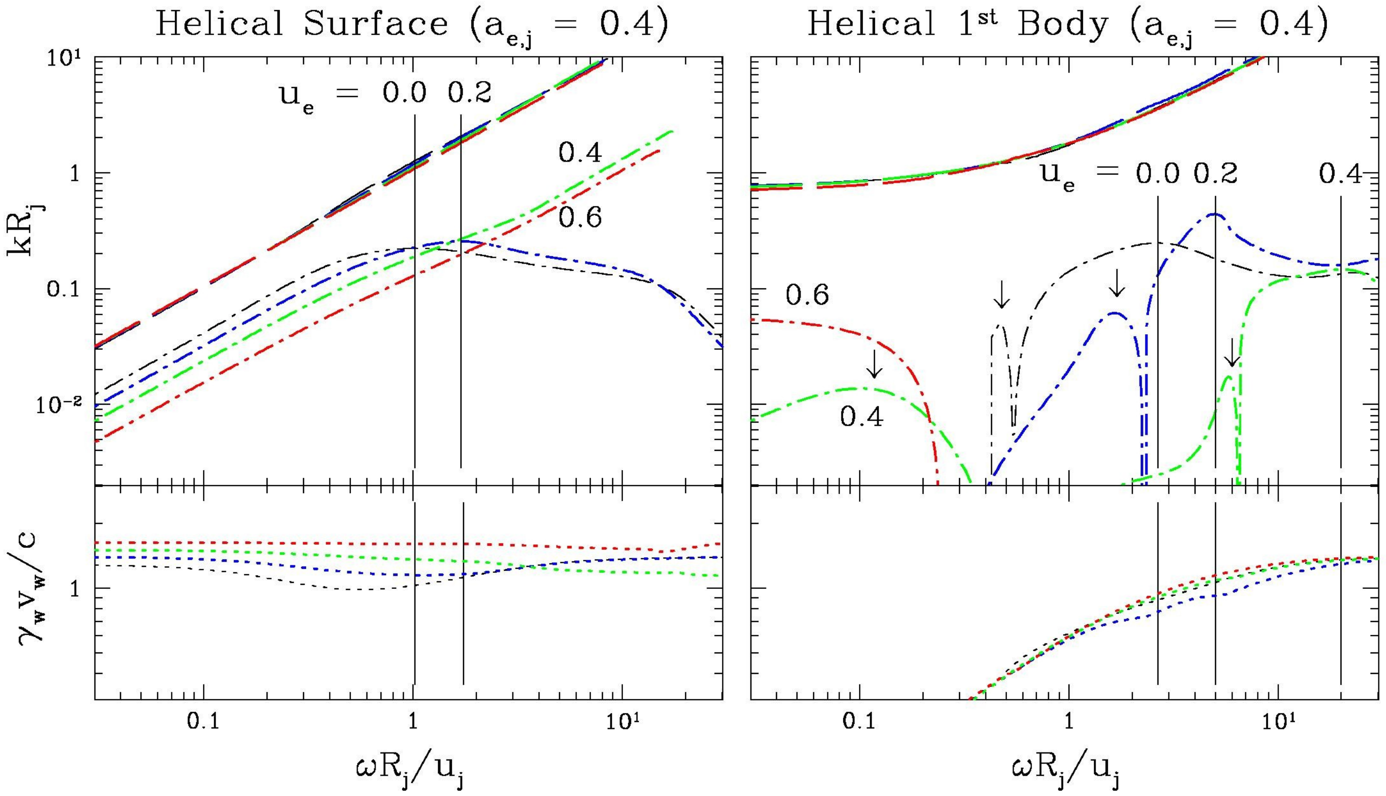
Growth Rate Reduction: Stability:

$$\gamma_j^2\gamma_e^2\left(u_j - u_e\right)^2 \longrightarrow \gamma_{Aj}^2\gamma_{Ae}^2\left(W_j/\gamma_{Ae}^2 + W_e/\gamma_{Aj}^2\right)\frac{B_j^2 + B_e^2}{4\pi W_j W_e}$$

$$\gamma_j^2\gamma_e^2\left(u_j - u_e\right)^2 < \gamma_{Aj}^2\gamma_{Ae}^2\left(W_j/\gamma_{Ae}^2 + W_e/\gamma_{Aj}^2\right)\frac{B_j^2 + B_e^2}{4\pi W_j W_e}$$

Stability Properties of a 3D RMHD Relativistic Jet Spine & Sheath

Dispersion Relation Numerical Solution
Effect of Sheath Flow on a Fluid Jet



Jet speed: $u_j = 0.916$ c **Sound speeds:** $a_j = a_e = 0.4$ c

Surface mode: growth rates (dash-dotted lines) reduced as sheath speed increases from $u_e = 0$ to 0.3 c.

Resonance: disappears for sheath speed $u_e > 0.35$ c

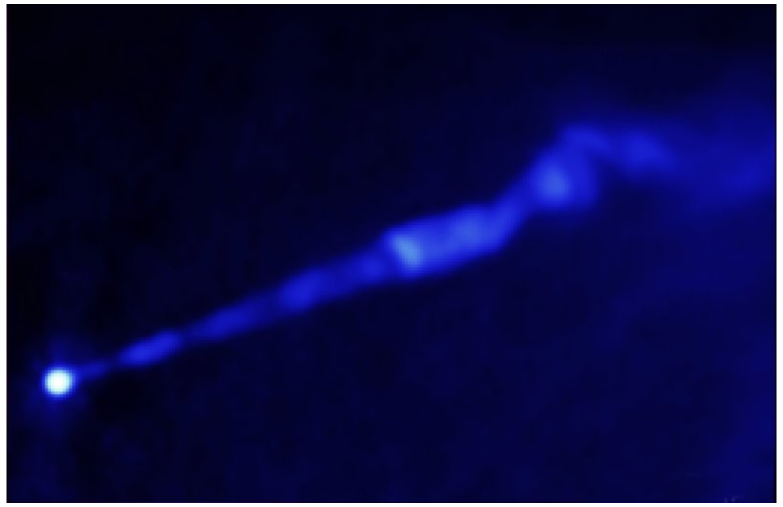
Body mode: downwards arrows indicate damping peaks

$\omega R_j/u_j >> 1$: damping for sheath speed $u_e > 0.5$ c

$\omega R_j/u_j << 1$: growth for sheath speed $u_e > 0.5$ c

M87 Jet: Spine-Sheath Configuration?

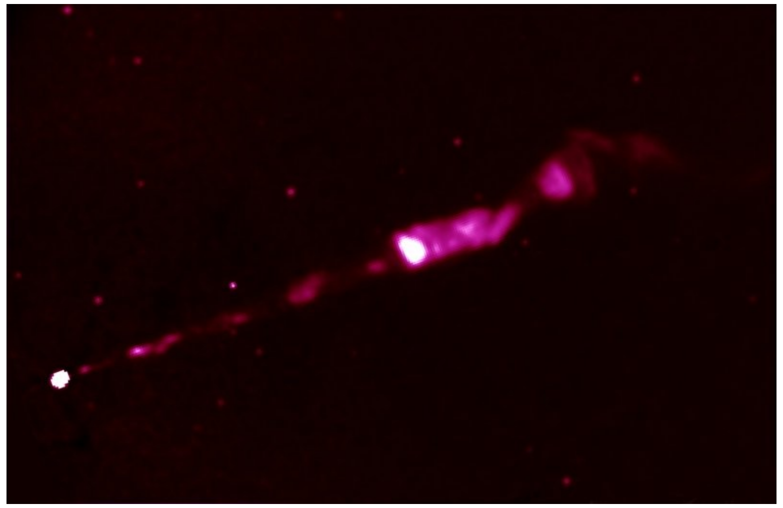
VLA Radio Image



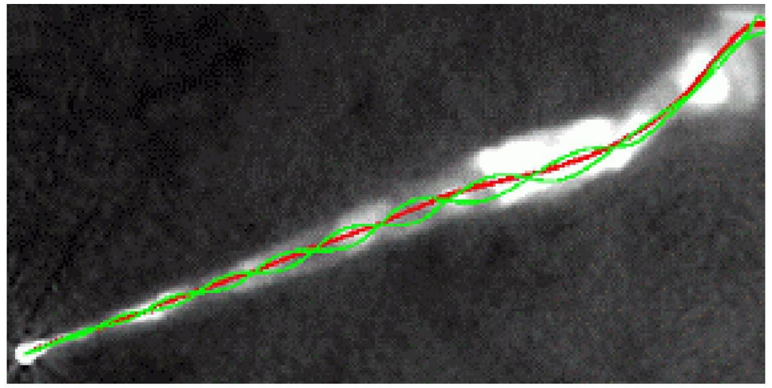
Jet Sheath ?
Typical Proper Motions < c
Biretta, Zhou, & Owen 1995

HST Optical Image

Jet Spine ?
Typical Proper Motions > c
Biretta, Sparks, & Macchetto 1999
Optical ~ inside radio emission



Jet Structures



Spine-Sheath interaction ?
Optical & Radio twisted filaments (green lines) &
helical twist (red line)
Lobanov, Hardee, & Eilek 2003

RMHD Simulations of Spine-Sheath Jet Stability

Key Questions of Jet Stability

- How do jets remain sufficiently stable?
- What are the Effects & Structure of CD/KH Instability?
- Can CD/KH Structures be linked to Jet Properties?

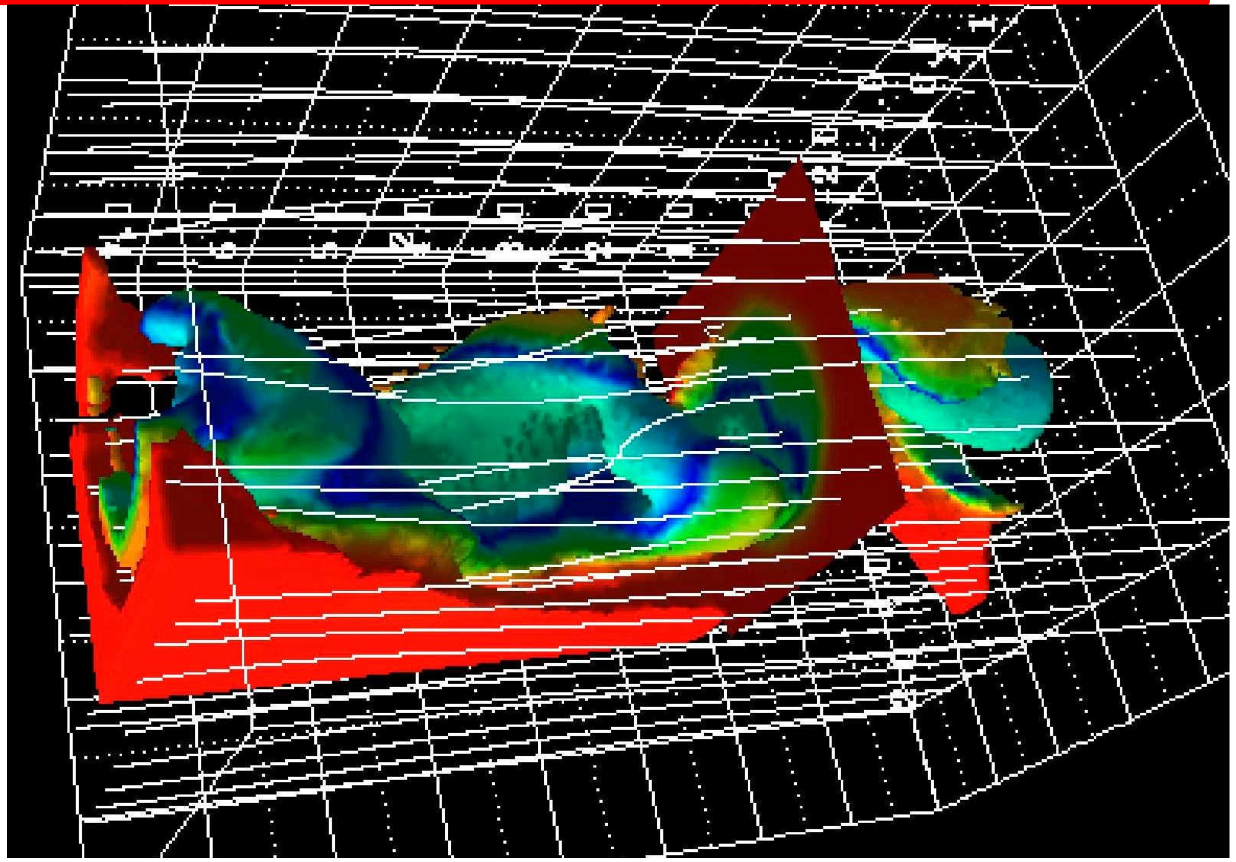
RMHD using RAISHIN

- Special relativity
- 3D Cartesian coordinate system
- Inflow & Outflow boundary conditions
- Divergence free magnetic fields

RMHD Simulation Initial Conditions

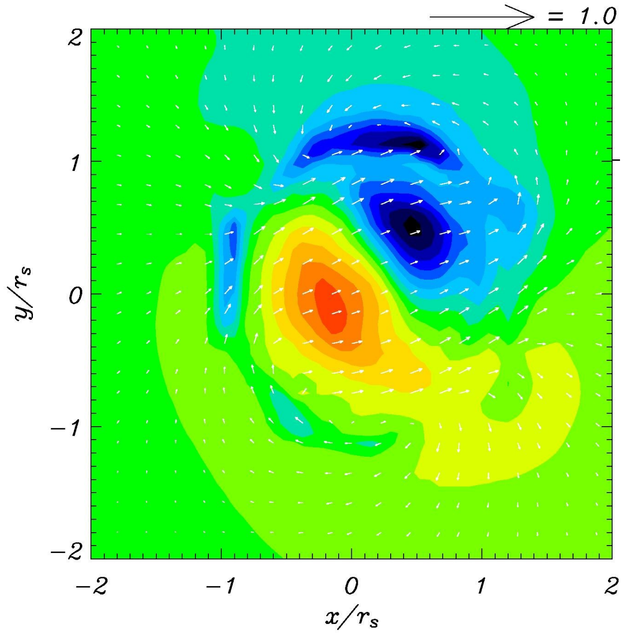
- Cylindrical Jet established across the computational domain
- Jet Lorentz factor = 2.5, $u_{jet} = 0.916$ c, $\rho_{jet} = 2\ \rho_{external}$
- External flow outside the jet, $u_{external} = 0$, c/2
- Jet precessed to break the symmetry
- RHD: $a_{jet} = 0.511$ c , $a_{external} = 0.574$ c, $v_{Alfven(j,e)} < 0.07$ c
- RMHD: $v_{Aj} = 0.45$ c, $v_{Ae} = 0.56$ c, $a_j = 0.23$ c , $a_e = 0.30$ c

Helically Twisted Pressure & Magnetic Structure

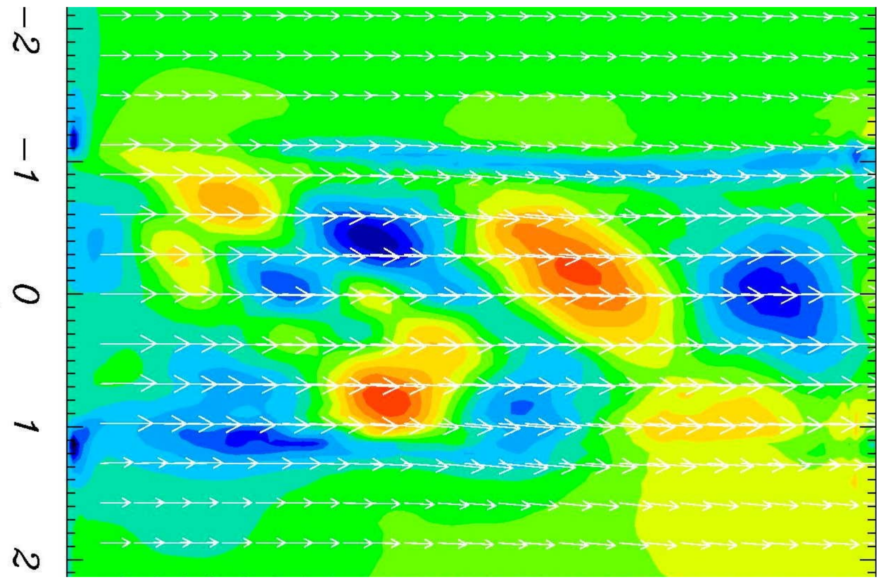


3D Rendering with B-field lines

Transverse cross section showing large scale helical pressure structure



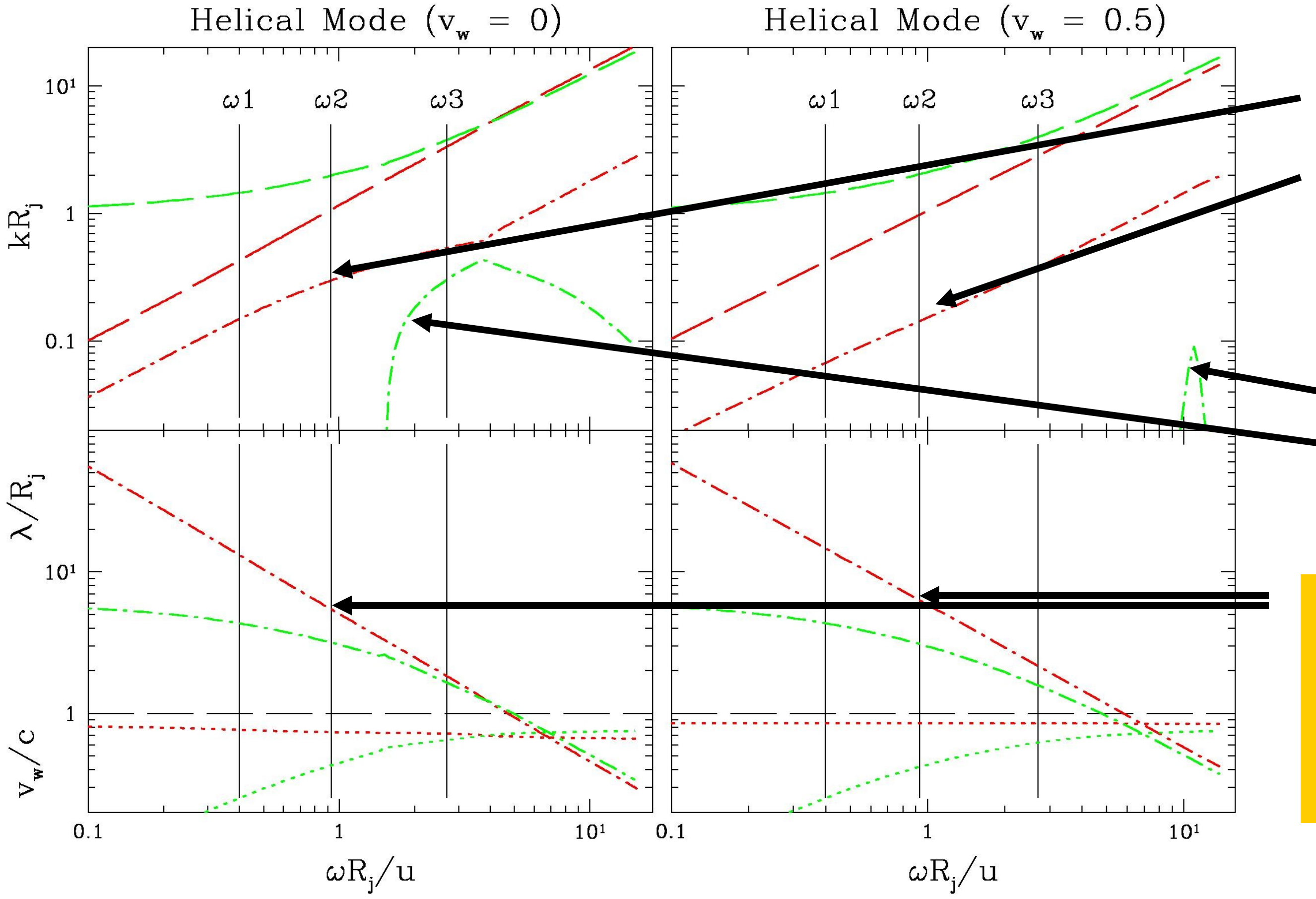
Longitudinal cross section showing small scale helical pressure structure



3D RHD Sheathed Jet Theory & Simulation Results

RHD Jet Dispersion Relation Solutions

Dispersion relation solutions kR_j as a function of $\omega R_j/u$. Dashed (dash-dot) lines indicate the real (imaginary) part of the wavenumber. Vertical lines indicate simulation precession frequencies. Wave speeds (dotted lines) and wavelengths (dash-dot lines) are shown below dispersion relation solutions.



Theory: Surface mode growth rate at ω2

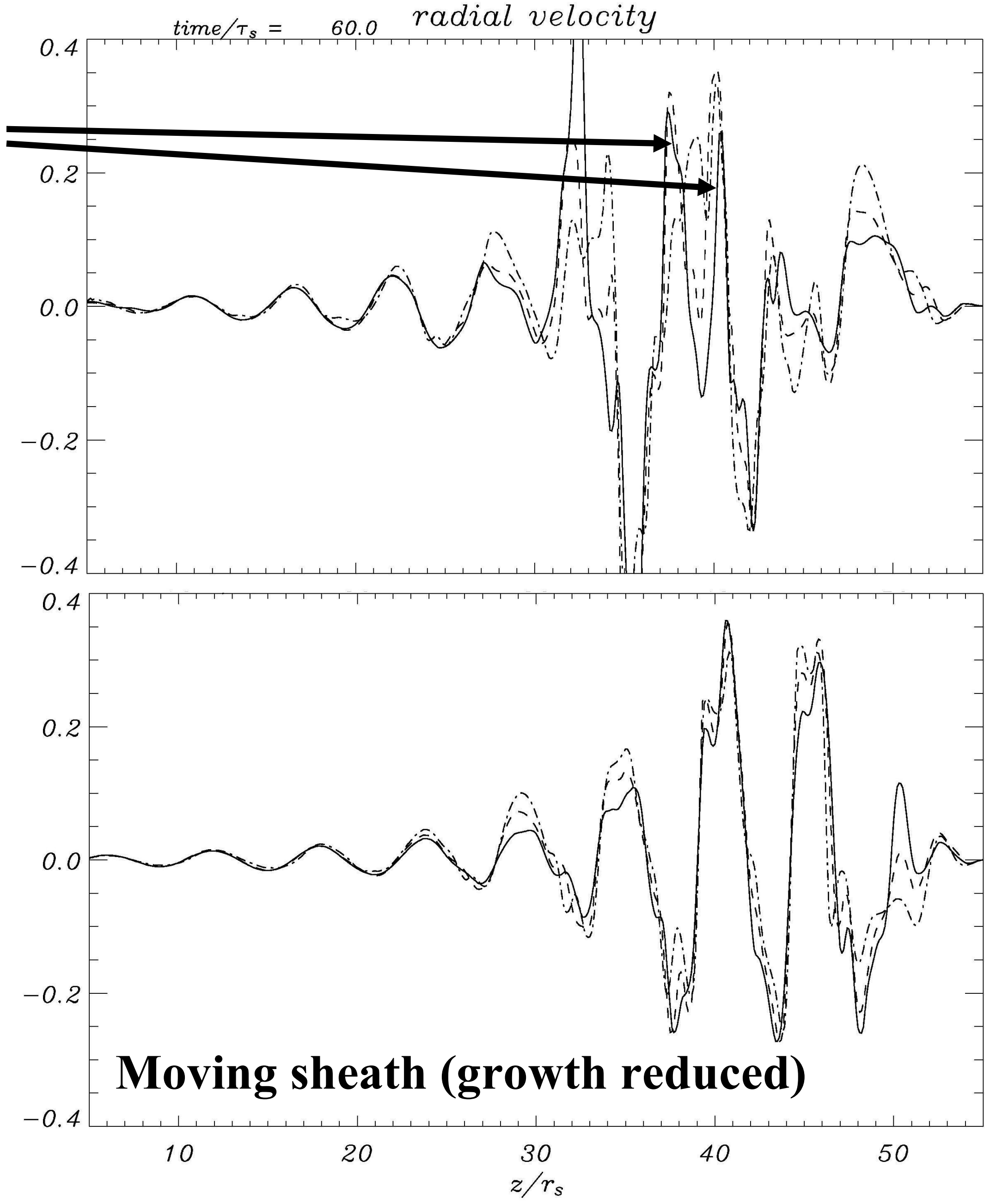
Theory: Body mode growth rate

Theory: Surface mode wavelength at ω2

Spatial growth of radial velocity:

stationary sheath (top) and moving sheath (bottom).

Simulation: Body mode substructure



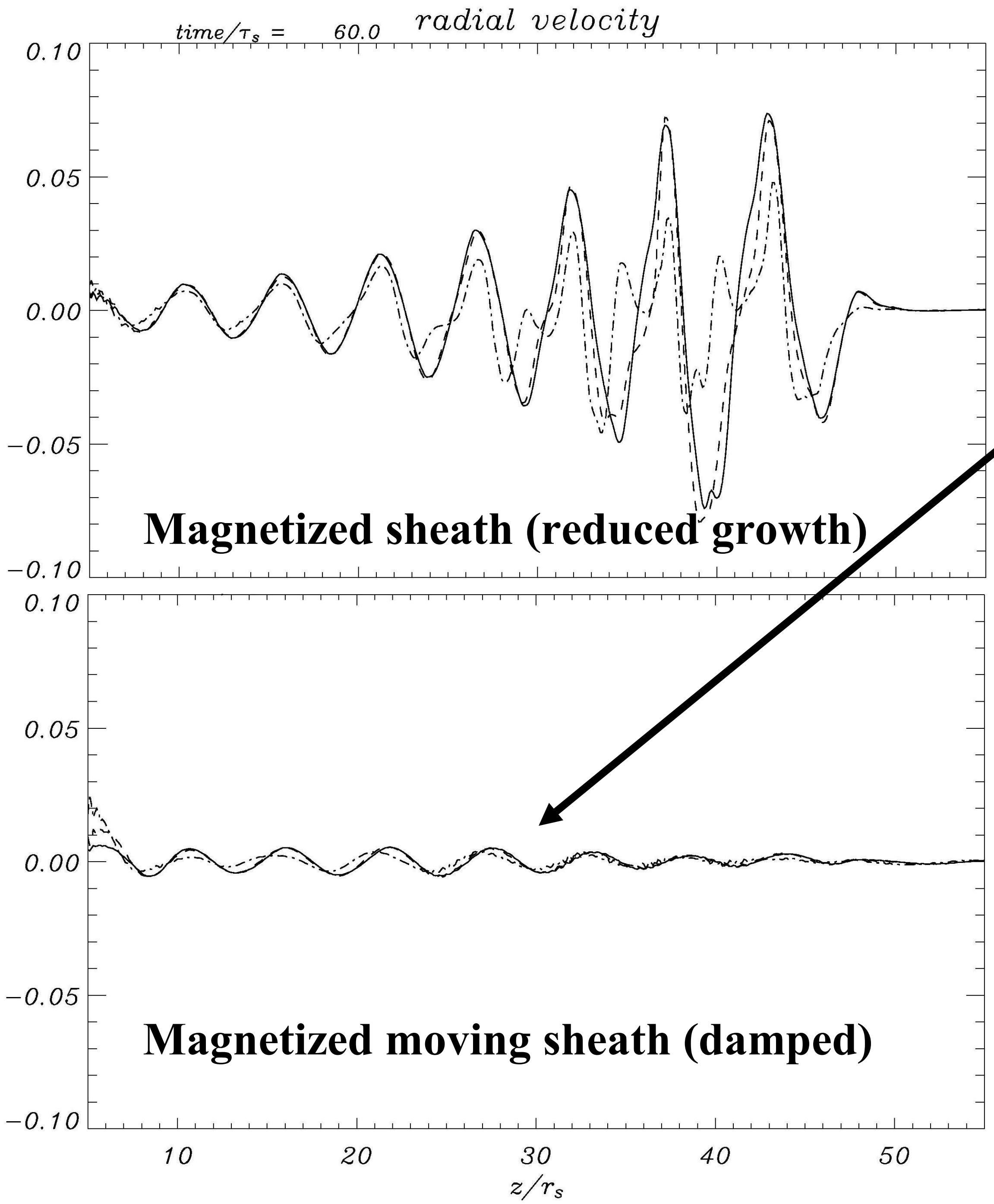
Moving sheath (growth reduced)

- A sheath with $v_w = c/2$ (right panels) significantly reduces the growth rate (red dash-dot) of the surface mode at simulation frequency ω2, and slightly increases the wavelength.
- Growth associated with the 1st helical body mode (green dash-dot) is almost eliminated by sheath flow.

- The moving sheath reduces the growth rate and slightly increases the wave speed and wavelength as predicted.
- Substructure associated with the 1st helical body mode is eliminated by sheath flow as predicted.

3D RMHD Sheathed Jet Simulation & Theory

Spatial growth of radial velocity:



Magnetized sheath (reduced growth)

Magnetized moving sheath (damped)

Panels show reduced growth relative to the fluid case for the magnetized sheath and damping in the presence of magnetized sheath flow.

Magnetized sheath flow has reduced the "velocity shear" to less than the "surface" Alfvén speed:

$$\boxed{\gamma_j^2\gamma_e^2\left(u_j-u_e\right)^2 < V_{As}^2}$$

Note that for comparable conditions in spine and sheath V_{As} = 2 γ_A $(B^2/4\pi W)^{1/2}$ and $(B^2/4\pi W)^{1/2}$ can be >> c.

Major Results

Growth of the KH instability driven by jet spine-sheath interaction is reduced significantly by mildly relativistic sheath flow and can be stabilized by magnetized sheath flow for spine Lorentz factors, γ_j, considerably larger than the Alfvén wave speed Lorentz factor, γ_A.

This result for axial magnetic field remains valid in the presence of an additional toroidal component. The crucial comparison is between the magnitude of the velocity shear and magnitude of the appropriate Alfvén speed projected on the wave vector, **k** (e.g., Hardee et al. 1992).

References

Biretta, J.A., Sparks, W.B., & Macchetto, F. 1999, ApJ, 520, 621
Biretta, J.A., Zhou, F., & Owen, F.N. 1995, ApJ, 447, 582
Hardee, P.E., Cooper, M.A., Norman, M.L., & Stone, J.M. 1992, ApJ, 399, 478
Lobanov, A., Hardee, P., & Eilek, J. 2003, NewAR, 47, 505