Modelling of the Particles Initial Energy Distribution with a Dynamical Model for FRII-type Radio Sources

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1. Introduction

Kaiser (2000, A&A, 362, 447; hereafter K2000) has constructed a 3-dimensional model of the synchrotron emissivity of the cocoon of FRII-type radio sources, and applied it into the self-simi3Alar model of the source's dynamics developed by Kaiser & Alexander (1997, MNRAS, 286, 215; KA) and its extension by Kaiser et at. (1997, MNRAS, 292, 723; KDA). K2000 has argued that careful account for the bulk backflow and energy losses, both radiative and adiabatic, can provide the spectral age comparable to the dynamical age. Blundell & Rawlings (2000, AJ, 119, 1111) have contended that this will be the case only if these ages are much less than 10 Myr.

Analysing the KDA model, we realized that in majority of extended FRII-type radio sources, even those without distorted lobe structures, the surface-brightness profiles are too far from a smooth shape predicted from the model, causing the fitted free parameters to be severely uncertain. On the other hand, the (K2000) method requires a high resolution observations of the radio lobes. But usually high-resolution observations of low-brightness sources cause a serious loss of the flux density, so that his method is limited to pretty strong sources like Cyg A.

We propose a method of the age determination based on a further exploitation of the KDA model that avoids the limitations present in the Kaiser's method. The KDA model has been succesively used by Machalski et al. (2004a, b, AcA, 54, 249, 391) to derive the basic physical parameters of FRII-type radio sources **at the known age**, namely the jet power, central density of the galaxy nucleus, energy density and pressure in their lobes, and the total energy deposited in the sources. The values of all these parameters were derived from fitting the model free parameters to the observed parameters of the source: its redshift, monochromatic radio luminosity at 1.4 GHz, projected linear size, and the cocoon's axial ratio. Here we show that performing the above fitting procedure for **a few** observing frequencies, i.e. enlarging a number of the observables used by adding the radio spectrum data of the entire cocoon, the **dynamical age of a given FRII-type radio source can be reliably estimated**.

2. The model application

In Table 1 we summarize the observational and model parameters characterizing the source's cocoon and used in the presented calculations.

Table 1: Observational and model parameters of the source's cocoon (cf. the text)

	Parameter	Symbol	Dimension
Observational	redshift	z	[dimensionless]
parameters	projected linear size	D	[kpc]
derived	axial ratio	AR	[dimensionless]
from radio maps	observing frequency	ν	[MHz]
and spectrum	monochromatic luminosity	P_ν	[W Hz^{-1}sr^{-1}]
Model free	central core radius	a_0	[kpc]
parameters	exponent of density profile	β	[dimensionless]
(to be fixed)	exponent of initial energy distribution	p=2$\alpha_{\rm inj}$+1	[dimensionless]
	energy cut-offs	$\gamma_{i,min}$, $\gamma_{i,max}$	[dimensionless]
	adiabatic indices	Γ_c, Γ_B, Γ_x	[dimensionless]
	age of the source	t	[yr]
	jets' orientation angle	θ	[°]
Model parameters	jet power	$Q_{\rm jet}$	[W]
derived for given	central density	ρ_0	[kg m^{-3}]
values of age t,	cocoon pressure	p_c	[N m^{-2}]
$\alpha_{\rm inj}$ and $\gamma_{i,max}$	cocoon energy density	u_c	[J m^{-3}]
	total energy	$E_{\rm tot}$	[J]

Fitting the model to the observed parameters of a given source for a number of the age values t ascribed to this source, for each frequency we obtain a set of solutions, $Q_{\rm jet}(t)$, $\rho_0(t)$, which show, as one can expect, that if the source with the apparent size D and the total luminosity of its cocoon P_ν was young, $Q_{\rm jet}$ would be stronger and ρ_0 would be lower (the central density – thinner) than their values for the same source but being much older. An example of the resulting the $Q_{\rm jet}-\rho_0$ diagram is shown in Fig. 1

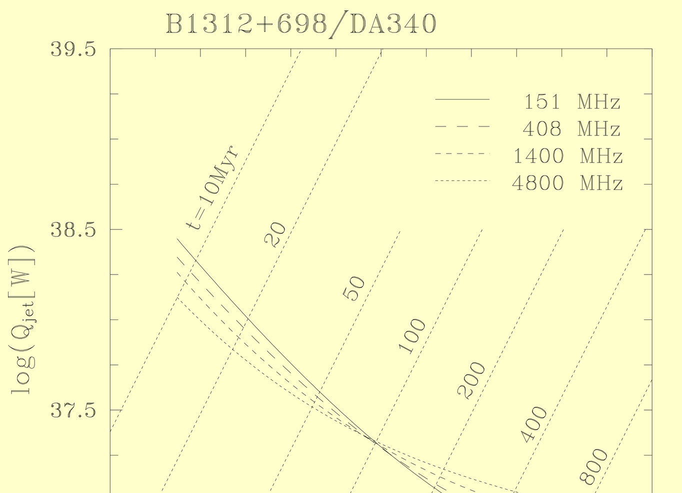

Figure 1: $Q_{\rm jet}-\rho_0$ diagram for the giant-size radio galaxy B1312+698. The intersection of the $Q_{\rm jet}-\rho_0$ curves gives the "age solution" of about 100 Myr if $\alpha_{\rm inj}$=0.53 and $\gamma_{\rm i,max}$=10^7.

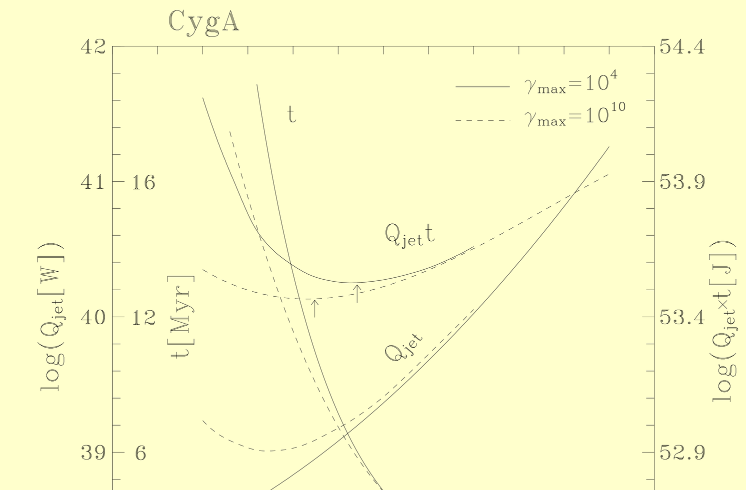

Figure 2: Diagram of jet power $Q_{\rm jet}$, age t, and energy delivered by the jet into the cocoon $Q_{\rm jet}t$ vs. $\alpha_{\rm inj}$ for the two (extremal) values of $\gamma_{\rm i,max}$. The arrows indicate $\alpha_{\rm inj}$ values for the relevant minimum kinetic energy. These $\alpha_{\rm inj}$ values correspond to the "best solution" for the age of Cyg A (cf. the text and Table 2)

This is clearly seen that the intersection of the $\log Q_{\rm jet}$–$\log\rho_0$ curves discriminates an age of the source and unique values of the jet power and the central density parameters. We mark this age as the "age solution". However the "age solution" depends on the fixed values of the model free parameters (cf. Table 1), especially on the parameters of the initial distribution of the particle energy, $\alpha_{\rm inj}$ and $\gamma_{\rm i,max}$ (cf. Fig. 1).

The dependence the "age solution" (hereafter marked as t) on the $\alpha_{\rm inj}$ and $\gamma_{\rm i,max}$ values is shown in Fig. 2. Besides the $t-\alpha_{\rm inj}$ diagram (for constant $\gamma_{\rm i,max}$), Fig. 2 shows the dependence of both the $Q_{\rm jet}$ and the product $Q_{\rm jet}\times t$ on $\alpha_{\rm inj}$. The latter is the kinetic energy of the jet delivered to the source. As the jet power $Q_{\rm jet}$ increases and the age decreases with increasing $\alpha_{\rm inj}$, the jet kinetic energy has a minimum. We call the "age solution" corresponding to the minimum as the "best solution", and argue that this "best solution" is the best estimate of the dynamical age of a given radio source.

The fitting procedure simply shows that if the real age of a given source was greater than the "best solution" age, the source would not be as luminous as it is observed. To provide the apparent luminosity at the observing frequencies, the jets' kinetic energy supplying the source must higher than the minimum energy. Although the "best solution" still depends on the value of $\gamma_{\rm i,max}$ parameter, we show this dependence introduces an uncertainty of the age of about 10 percent only.

Given the age determined for each of the sources analysed, not only the unique values of the jet power $Q_{\rm jet}$ and the central core density ρ_0 are determined. The "best solution" provides also unique values of the source (cocoon) energy density and pressure, u_c and p_c, respectively, as well as the total radiative energy $E_{\rm tot}$ (cf. Table 1).

The minimum of the jet kinetic energy (which gives the "best solution" of the age) corresponds to the equivalent minimum of u_c and p_c. The values of u_c as the function of age of the galaxy 3C294 is plotted in Fig. 3 for the four observing frequencies and the four $\alpha_{\rm inj}$ values. Again, each intersection of the u_c-t curves for the given $\alpha_{\rm inj}$ value gives the "age solution", and these "age solutions" have a minimum at $\alpha_{\rm inj}$=0.686. This minimum corresponds to the "best solution" for the age of 3C294, i.e. t=3.5 Myr (cf. Table 2).

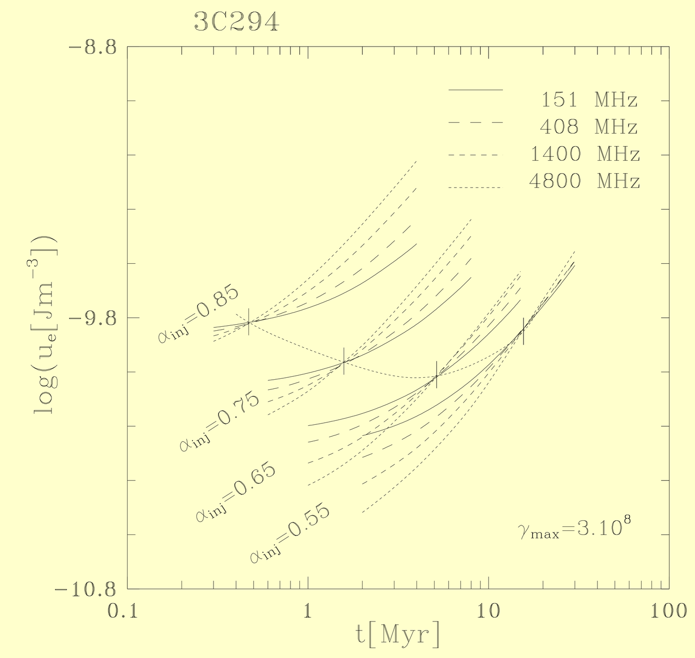

Figure 3: u_c-t diagram for 3C294. The energy densities determined by the intersections of the u_c curves fitted for four different values of $\alpha_{\rm inj}$, and each of them at the four observing frequencies are connected with an additional dotted curve. This curve has a minimum compatible with the minimum of the jet kinetic enegy. Both of these minima correspond to $\alpha_{\rm inj}$=0.686 and the age of 3.48 Myr (cf. Table 2)

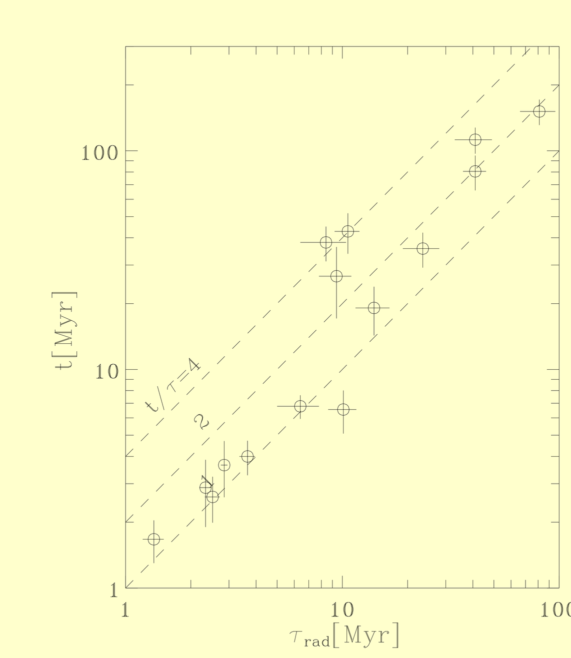

Figure 4: "Best solution" age vs. synchrotron age for the analysed radio galaxies

3. Exemplary results

The "best solutions" for the dynamical age of a number of FRII-type radio galaxies, and the corresponding values of $\alpha_{\rm inj}$ are given in Table 2. In order to show how these solutions depend on the assumed value of the $\gamma_{\rm i,max}$ parameter, we give the "best solution" for its three values: 10^7, 3 10^8, and 10^{10}. Column 8 of Table 2 gives the ratio of the standard deviation of the age (in columns 3, 5, and 7) and the mean of age. It shows that the age estimates ("best solutions") differ from ∼4% to ∼30%, mostly by ∼10% only.

A quality of the dynamical age estimated with the method presented can be justified by its comparison with the synchrotron (radiative) age derived from the spectral ageing analysis (e.g. Myers & Spangler 1985, ApJ, 291, 52; Carilli et al. 1991, ApJ, 383, 554). In this analysis the radiative age of emitting particles is determined from the "break" frequency in the observed radio spectrum and the magnetic field strength, usually computed under the equipartition conditions. In order to find this "break" frequency, the $\alpha_{\rm inj}$ value must be known. In most of the published papers concerning the above analysis, the value of $\alpha_{\rm inj}$ was usually identified with the slope of the observed low-frequency spectrum. Because of a reasonable criticism of this simple method (cf. Introduction), Murgia (1996, Laurea Thesis, Univ. Bologna) developed the software SYNAGE which allows the best fit of the spectral data to the theoretical models of the energy losses by constraining the values of the most important model parameters, especially a value of $\alpha_{\rm inj}$.

The $\alpha_{\rm inj}$ value for the C.I. model, $\alpha_{\rm CI}$, fitted to the spectrum of the lobes of the analysed sources using the SYNAGE algorithm, is given in column 9 of Table 2. It is easy to see that the value of $\alpha_{\rm CI}$ is mostly higher than the values of $\alpha_{\rm inj}$ determined by the minimum of the jet energy, though the errors in $\alpha_{\rm inj}$ are of the same order as the errors in $\alpha_{\rm CI}$ (also given in column 9). Only for Cyg A and (formally) for 3C165 the fitted values of $\alpha_{\rm CI}$ are lower than the values of $\alpha_{\rm inj}$ in columns 2, 4 and 6, and $\alpha_{\rm CI}\approx\alpha_{\rm inj}$ for 3C55.

Table 2: $\alpha_{\rm inj}$ values for the minimum kinetic energy of the jet and the corresponding "best solution" of age for the analysed sample of FRII-type radio galaxies. Column 8 gives the ratio of the standard deviation to the mean of the ages in columns 3, 5 and 7. The columns 9 and 10 give $\alpha_{\rm inj}$ value fitted for the C.I. model using the SYNAGE package of Murgia, and the synchrotron (radiative) age of the sources compiled from the literature, respectively.

	$\gamma_{\rm max}$=10^7		$\gamma_{\rm max}$=3 10^8		$\gamma_{\rm max}$=10^{10}				
Source	$\alpha_{\rm inj}$	t[Myr]	$\alpha_{\rm inj}$	t[Myr]	$\alpha_{\rm inj}$	t[Myr]	$\Delta t/\bar{t}$	$\alpha_{\rm CI}$	$\tau_{\rm rad}$[Myr]
1	2	3	4	5	6	7	8	9	10
Cyg A	0.607	6.78	0.618	5.94	0.621	5.73	0.09	0.571±.023	6.4±1.4
3C55	0.628	26.7	0.632	24.1	0.634	22.9	0.08	0.630±.060	9.4±1.6
3C103	0.576	42.8	0.589	36.5	0.595	33.9	0.12	0.617±.099	10.6±1.4
3C165	0.557	80.5	0.580	64.5	0.589	59.1	0.16	0.523±.051	41±5
3C239	0.673	2.88	0.678	2.72	0.686	2.67	0.04	0.716±.033	2.34±0.15
3C247	0.546	1.67	0.570	1.31	0.581	1.17	0.19	0.599±.021	1.35±0.15
3C280	0.560	2.61	0.575	2.22	0.583	2.00	0.14	0.694±.031	2.52±0.2
3C292	0.566	38.1	0.577	36.0	0.582	35.0	0.04	0.782±.049	8.4±2
3C294	0.683	3.65	0.686	3.48	0.688	3.37	0.04	0.898±.067	2.85±0.1
3C322	0.557	4.00	0.569	3.93	0.577	3.73	0.04	0.624±.041	3.65±0.3
3C330	0.543	6.55	0.563	5.26	0.572	4.77	0.17	0.580±.039	10.1±1.5
3C332	0.528	35.7	0.555	26.1	0.564	23.2	0.23	0.620±.040	23.5±4.5
B0908+376	0.514	19.1	0.543	12.6	0.553	10.8	0.31	0.548±.082	14±2.5
B1209+745	0.512	151.3	0.539	128.8	0.550	120.1	0.12	0.764±.042	81±15
B1312+698	0.506	112.3	0.536	88.8	0.548	81.5	0.17	0.639±.035	41±8

A discrepency between the values of $\alpha_{\rm CI}$ and $\alpha_{\rm inj}$ for the other sources can be explained either by (i) a lack of the flux density data at the frequencies below ∼100 MHz (which may decide about a proper determination of the slope of the initial spectrum) and the uncertainties of the available data, or by (ii) an influence of other physical processes changing the apparent radio spectrum and not taken into account in the KDA model, like reacceleration of the relativistic particles, evolution of the local magnetic fields, etc.

Finally, the synchrotron age with its error for the sources in Table 2, $\tau_{\rm rad}$, collected from the papers of Alexander & Leahy (1987, MNRAS, 225, 1); Leahy et al. (1989, MNRAS, 239, 401): Liu et al. (1992, MNRAS, 257, 545): and Parma et al. (1999, A&A, 344, 7) is given in column 10 of Table 2. In accordance with the suggestion of Blundell & Rawlings, the synchrotron ages and our dynamical age estimates are comparable for the young sources, i.e. those whose ages are less or much less than 10 Myr. For the largest sources in Table 2, the ratio of the dynamical age estimate and the synchrotron age rises to the values from ∼2 to ∼4 in the case of 3C292, the high-redshift giant radio galaxy. This conclusion agrees with the earlier result on that ratio determined for the three giant radio galaxies: J0912+3510, J1343+3758, and J1451+3357 (Jamrozy & Machalski 2005, Baltic. Astr. 14, 381). In Fig. 4, the "best solution" age estimate is plotted vs. the synchrotron age.

4. Theses and conclusions

– There is not a canonical value of the $\alpha_{\rm inj}$ parameter good for all the sources. In order to estimate the dynamical age of a particular FRII-type radio source, the best $\alpha_{\rm inj}$ value is determined by the minimum of the jet kinetic energy which can be found with the fitting procedure.

– In some radio sources, mostly at high redshifts, the best $\alpha_{\rm inj}$ value is higher than the values permissible by the theory of relativistic shock propagation and compression, as well as the Fermi-acceleration of high-energy particles (cf. Heavens & Drury 1988). This may suggest an influence of other physical processes changing the apparent radio spectrum and not taken into account in the KDA model, like reacceleration of the relativistic particles, evolution of the local magnetic fields, etc.

– The $\alpha_{\rm inj}$ values identified with the observed low-frequency spectral indices are frequently too high, causing the source's age is underestimated and the implied mean expansion velocity is evidently too high. For example, the $\alpha_{\rm CI}$=0.898±0.067 fit to the spectrum of the high-redshift radio galaxy 3C294, implies its age of 0.25 Myr and the expansion velocity 0.880c!

– Our "best solution" ages and the synchrotron ages are comparable if these ages are less than about 10 Myr. For older sources the ratio of $t/\tau_{\rm rad}$ may rise to the values of 2 – 4.