Simulation of relativistic jets near a spinning black hole

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Abstract

It is generally believed that the magnetic field plays a key role for relativistic jet generation in the vicinity of black holes. In comparison with stationary processes which were suggested during the last few decades we developed a non-stationary mechanism which is build on the interaction of a spinning black hole with a magnetic flux tube/string. Using the Lagrangian approach we can consider the magnetized plasma as a set of flux tubes/strings that allow us to simplify the problem significantly. Numerical simulation demonstrated how the energy can be extracted from a Kerr black hole and showed the process of relativistic jet formation. A magnetic flux tube falling into a black hole is involved into the differential rotation around the hole. This leads to a string braking and to the generation of negative energy and momentum. According to conservation laws, positive energy and momentum is generated and redistributed along the string extracting the spinning energy from the black hole. This is a non-local variant of the Penrose process, but now we don't need to consider the particles decay. The energy extraction is accompanied by the relativistic jet production. The parts of the flux tube with abundance of energy and momentum leave the ergosphere producing a helical jet structure which is propagating along the rotation axis with relativistic speed. This continuous process leads to a strong stretching of the flux tube: one part of the string with negative energy is fixed inside the ergosphere whereas another one moves away from the black hole with relativistic speed. Such behavior of the string inevitably leads to the reconnection process which changes the magnetic flux tube topology and releases Maxwellian tensions. The reconnection gives rise to a plasmoid (part of the string which is separated from the initial string), which moves away from the black hole carrying energy and momentum. The described process is repeated, so the jet structure is composed of a chain of plasmoids which are propagating along the spin hole axis.

Basic equations

A convenient mathematical formulation for studying the behaviour of flux tubes can be obtained through the introduction of Lagrangian coordinates into the relativistic magnetohydrodynamic equations (RMHD) of general relativity. The RMHD equations are (Lichnerowicz, 1967):

$$\nabla_i \rho u^i = 0 \tag{1}$$

$$\nabla_i (h^i u^k - h^k u^i) = 0 \tag{2}$$

$$\nabla_i T^{ik} = 0 \tag{3}$$

Here (1) – continuity equation, (2) – Maxwell's equation, (3) energy-momentum equation. Here u^i is the time-like vector of the 4-velocity, $u^i u_i = 1$, $h^i = {}^*F^{ik} u_k$ is the space-like 4-vector of the magnetic field, $h^i h_i < 0$, ${}^*F^{ik}$ is the dual tensor of the electromagnetic field, and T^{ik} is the energy-momentum tensor.

$$T^{ik} = Qu^i u^k - Pg^{ik} - h^i h^k/4\pi \tag{4}$$

$$P \equiv p - h^i h_k/8\pi \tag{5}$$

$$Q \equiv p + \varepsilon - h^i h_k/4\pi \tag{6}$$

Here p is the plasma pressure, P is the total (plasma + magnetic) pressure, e is the internal energy including ρc^2, g^{ik} is the metric tensor with signature (1,-1,-1,-1). Generally speaking, $\nabla_i h^i = 0$, but we can find a function q such that $\nabla_i q h^i \neq 0$. Then using (1) the Maxwell equation (2) can be rewritten in the form of a Lie derivative:

$$\frac{h^i}{\rho} \nabla_i \frac{u^k}{q} = \frac{u^i}{q} \nabla_i \frac{h^k}{\rho} \tag{7}$$

and we can therefore introduce "frozen-in" coordinates, such that:

$$\frac{\partial x^i}{\partial \tau} = \frac{u^i}{q}, \quad \frac{\partial x^i}{\partial \alpha} = \frac{h^i}{\rho} \tag{8}$$

with new coordinate vectors u^i/q, h^i/ρ tracing the trajectory of a fluid element and the magnetic field in a flux tube. Using (8) the energy-momentum equation (3) can be rearranged to form a set of string equations in terms of the frozen-in coordinates:

$$-\frac{\partial}{\partial \tau}\left(\frac{Qq}{\rho}\frac{\partial x^i}{\partial \tau}\right) - \frac{Qq}{\rho}\Gamma^l_{ik}\frac{\partial x^i}{\partial \tau}\frac{\partial x^k}{\partial \tau} + \tag{9}$$

$$+\frac{\partial}{\partial \alpha}\left(\frac{\rho}{4\pi q}\frac{\partial x^i}{\partial \alpha}\right) + \frac{\rho}{4\pi q}\Gamma^l_{ik}\frac{\partial x^i}{\partial \alpha}\frac{\partial x^k}{\partial \alpha} = -\frac{g^{il}}{\rho q}\frac{\partial P}{\partial x^i}$$

here $q = 1/(g_{ik} x_\tau^i x_\tau^k)^{1/2}$, and Γ_{ik}^l is the Christoffel symbol.

Nonlinear string in Kerr metric

The Kerr metric in Boyer-Lindquist coordinates is given by the following line element (Misner at al. 1973):

$$ds^2 = \left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 - \tag{10}$$

$$-\left(r^2 + a^2 + \frac{2Mra^2}{\Sigma}\sin^2\theta\right)\sin^2\theta d\phi^2 + \frac{4Mra}{\Sigma}\sin^2\theta d\phi\, dt$$

$$\Delta = r^2 - 2Mr + a^2, \; \Sigma = r^2 + a^2\cos^2\theta$$

Here M and a are the mass and angular momentum of the hole, respectively, and we have used a system of units in which c = 1, G = 1. Let us now consider a test flux tube, which falls from infinity into a Kerr black hole. For cyclic variables t and φ, the energy and angular momentum conservation laws for the flux tube can be written as:

Results of the numerical simulation

In the spinning Kerr geometry the leading part of the falling flux tube progressively loses angular momentum and energy as the string/tube brakes, which leads to the creation of negative energy inside the ergosphere (fig. 1a). To conserve energy and angular momentum for the tube as a whole, the positive energy and angular momentum has to be generated for the trailing part of the tube. This is a variant of the Penrose process (Penrose, 1969).

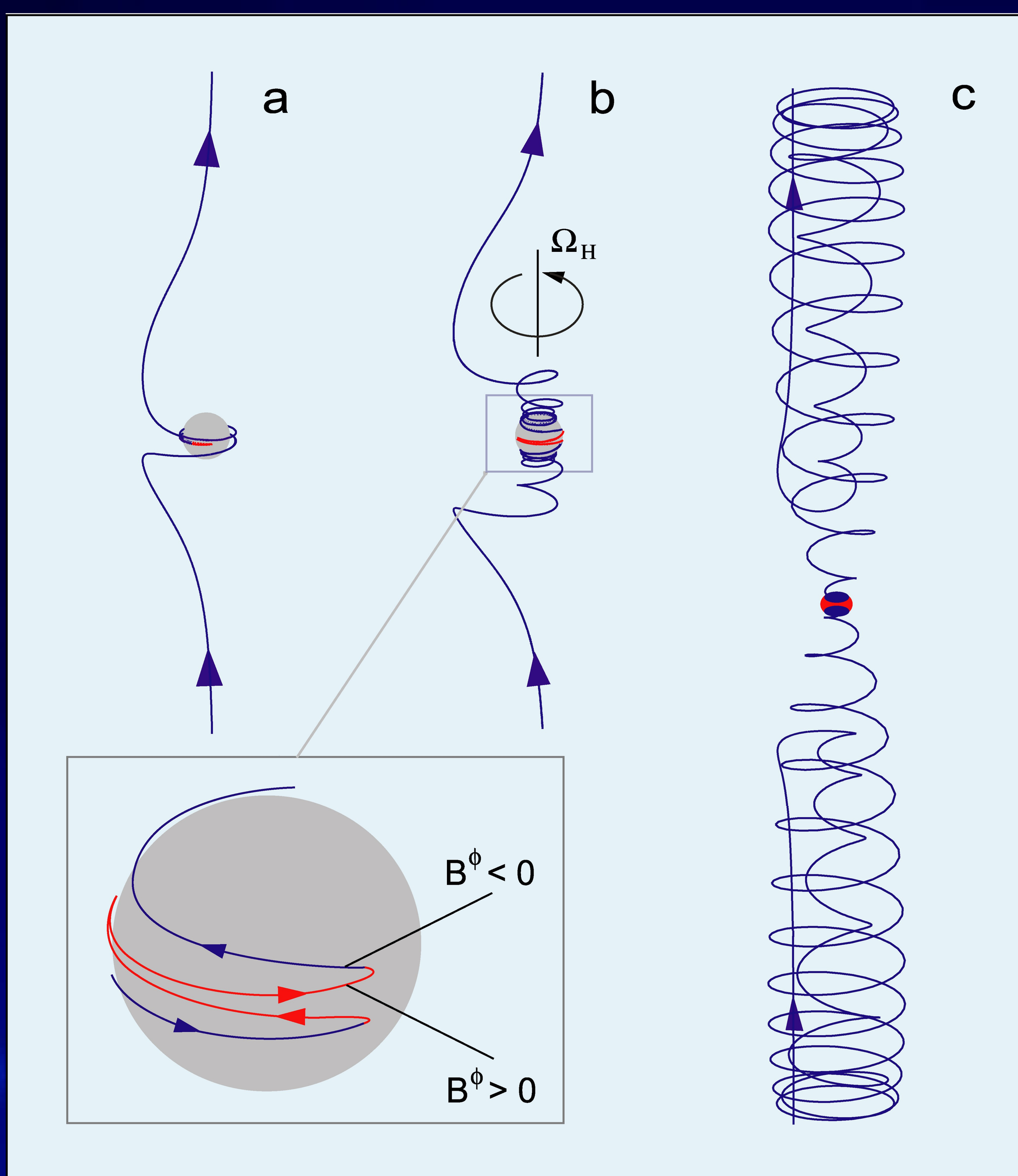

Fig. 1 This figure shows the different moments of simulations. (a) – corresponds to the negative energy creation onset (part of the string with negative energy labeled by red color), (b) – beginning of the spiral structure creation, (c) – the last moment of simulation, (d) – magnetic flux tube structure near event horizon.

For the first "confinement" stage both parts with negative and most of the positive energy are localized in the narrow layer near the event horizon (V.S. Semenov, S.A. Dyadechkin, I.B. Ivanov and H.K. Biernat, 2002). Then, energy and momentum are extracted from the ergosphere in the form of a spiral wave which is a relativistic jet (V.S. Semenov at al., 2002, 2004).

The continuous stretching of the flux tube is accompanied by the jet creation, evidently can't last infinitely long. The physical process which can change string topology and releases Maxwellian tensions is magnetic *reconnection*. It leads to the creation of a plasmoid propagating along the black hole rotation axis and carrying energy and angular momentum far away from the black hole.

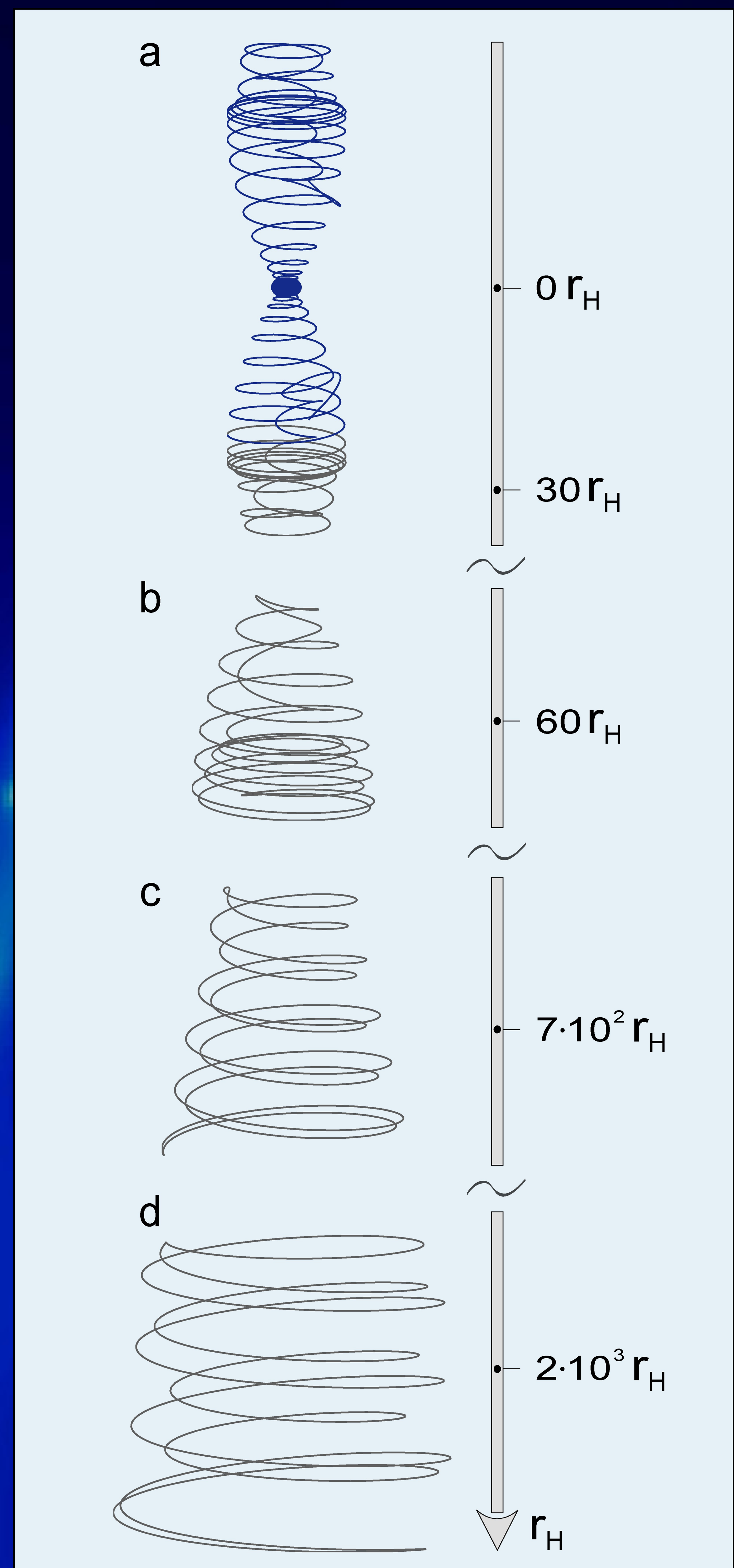

Fig. 2 Magnetic flux tube stretching can't last infinitely long. Topology of the string can be changed by magnetic reconnection. This figure shows the result of a numerical simulation of reconnection process: (a) represents the initial stage of reconnection (gray color marks the part of the string which is separated from jet), (b), (c) and (d) represent the development of a separated flux tube in the course of time. The result of reconnection is a plasmoid which carries energy and angular momentum far away from the black hole (d).

The comparison of flux tube and cosmic string behaviors

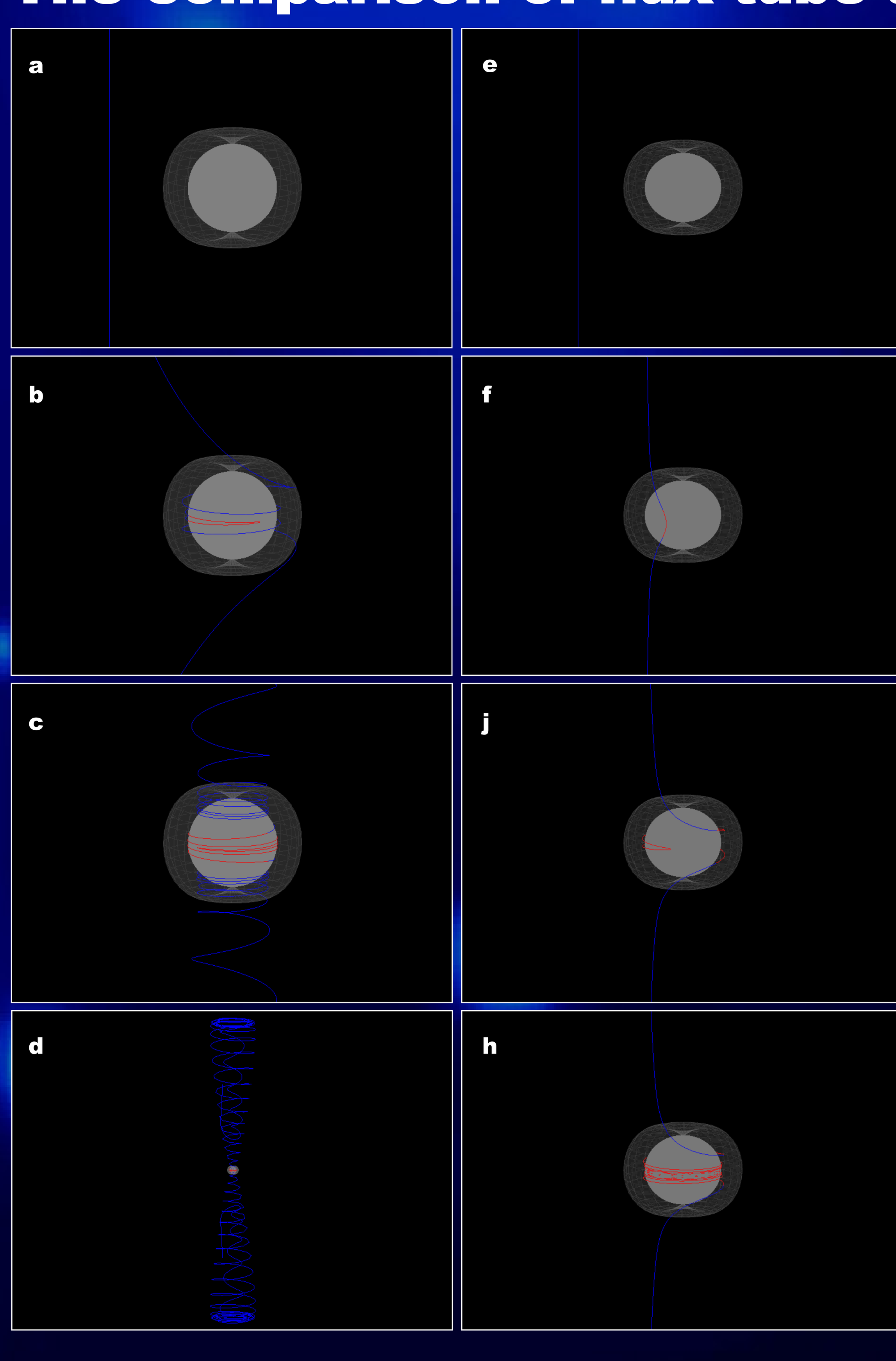

Fig. 3 Snapshots of different moments of magnetic flux tube (a1…a4) and cosmic string (b1…b4) behavior in the vicinity of a Kerr black hole. Red color labels a part of the flux tube/string which gains the negative energy. Full movies of flux tube and cosmic string behavior are available on geo.phys.spbu.ru/~ego/flux_tube and geo.phys.spbu.ru/~ego/cosmic_string

The cosmic string behavior (generated by the Numbu-Goto action) at an early stage is similar to that of the flux tube. Involved in differential rotation, the central part of the cosmic string starts to lose energy and angular momentum by means of string braking (Fig. 3e,f). Stretching and twisting around the event horizon, the central part of the string gains negative energy in the ergosphere (Fig. 3f,j). To compensate this losses, positive energy is generated and extracted from the ergosphere as in the flux tube case (Fig. 3d). Unfortunately, increasing of numerical errors near the event horizon break down the simulation and we can observe only initial stages of negative energy creation (Fig. 3h). Within the frame of direct analogy with the magnetic flux tube, our results show the very beginning of cosmic string jet formation in Kerr geometry.

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