Time-dependent relativistic reconnection of strong magnetic fields in symmetric current sheet.

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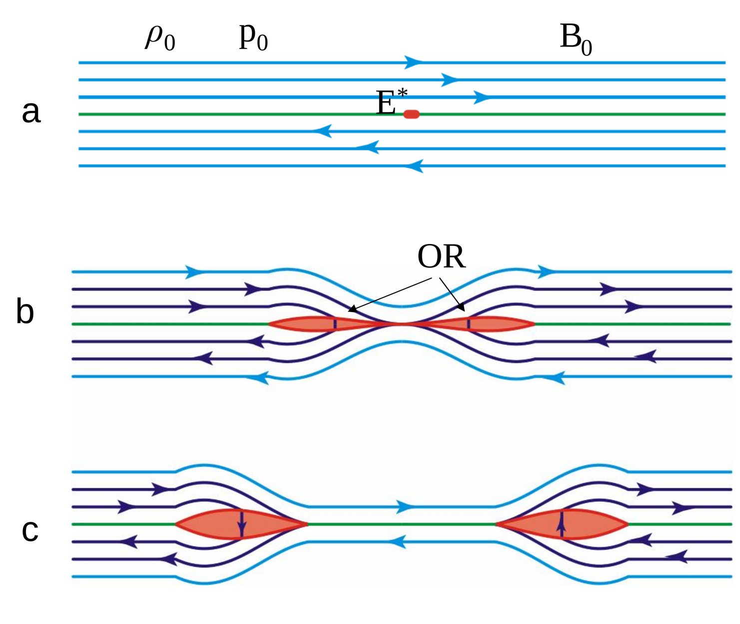
Abstract. The role of a magnetic reconnection is well known in magnetosphere and solar physics where it serves as a basis for magnetosphere substorms and solar flares. There are many astrophysical objects such as pulsars, magnetars, black holes, etc., where the reconnection of much stronger magnetic fields comparing with solar fields has to be important as well. Therefore development of the relativistic reconnection theory seems to be significant importance. The model of unsteady Petschek-type reconnection of arbitrary strong magnetic fields is presented where all necessary relativistic effects are taken into account. The reconnection is initiated due to local enhancement of plasma resistivity inside the diffusion region, which results in appearance of an electric field along the X-line. This electric field (so called, the reconnection rate) is considered as a given arbitrary function of time and then all MHD parameters as well as a shape of moving Petschek-type shocks are obtained from the relativistic MHD equations. It was shown that plasma is accelerated at the slow shocks to ultrarelativistic velocities with high Lorentz-factors only for current layers embedded into strong magnetic fields and cold plasma. In this case the plasma is strongly compressed and heated while the normal size of the outflow region with the accelerated plasma becomes very small.

Reconnection starts with abrupt drop of plasma conductivity in a small part of current sheet, so called, diffusion region. As a result electric field E^* is generated and is transferred by MHD waves from the diffusion region to the system at large (Fig. a) which leads to decay of the disturbed part of the current sheet into a system of slow shocks. Plasma is highly accelerated and heated at the shock fronts forming the outflow region (OR) with strong plasma flow and weak magnetic field (Fig. b). At some stage the reconnection must switch-off, then the outflow regions detach from the site where the electric field was initiated, and propagate along the current sheet as a solitary waves (Fig. c).



Statement of the problem.

- Coordinates: $x^1 = x$ is tangential to the current sheet, $x^3 = z$ is normal.
- Initial current sheet: density ρ_0 ,pressure p_0 and oppositely directed magnetic fields $h^i = (0, \pm B_0, 0, 0)$.
- Ideal plasma is considered as a polytropic gas with the following relation between a specific enthalpy, pressure and density: $w = c^2 + p\gamma/(\gamma-1)\rho$; $p = p_0(\rho/\rho_0)^\gamma$ where $\gamma>1$ is a polytropic exponent.
- Electric field $E^*(x^0)$ along the reconnection line is considered as given function.
- Approximation of weak reconnection $E^*/E_A << 1$ $(E_A = v_a B_0/c)$
  We have to find a solution of RMHD equations as well as shock relations, and the shape of moving slow shocks using initial parameters of the current layer and the electric field $E^*(x^0)$.

The relativistic magnetohydrodynamic (RMHD) equations

$\partial_k T^{ik} = 0$ — the equation of energy-momentum conservation

$\partial_k\left(h^k u^i - h^i u^k\right) = 0$ — the magnetic induction equation

$\partial_k(\rho u^k) = 0$ — the continuity equation

where u^i is a 4-vector of velocity and $h^i = {}^*F^{ik} u_k$ is a 4-vector of magnetic field, a energy-momentum tensor is

$$T^{ik} = \left(\rho w - \frac{1}{4\pi} h^i h_i\right) u^i u^k - \left(p - \frac{1}{8\pi} h^i h_i\right) g^{ik} - \frac{1}{4\pi} h^i h^k$$

with g_{ik} is the metric tensor with signature (1,-1,-1,-1)

Relations across an MHD discontinuity in relativistic plasma can be presented as follows:

$$\{T^{ik}\} n_k = 0$$
$$\{\rho u^k\} n_k = 0$$
$$\{h^i u^k - h^k u^i\} n_k = 0$$

where n_k is a 4-normal vector to shock wave surface.

Solution in the outflow region

The pressure and the density

$$p = p_0 + \frac{B_0^2}{8\pi} \qquad \rho = \frac{4\pi\rho_0^2}{\gamma-1} \frac{c_s^2 + \gamma v_a^2/2}{\sqrt{4\pi Q\left(4\pi Q - B_0^2\right) - 4\pi\rho_0 c^2}}$$

the plasma velocity

$$u^k = \left(\left(\frac{4\pi Q}{4\pi Q - B_0^2}\right)^{1/2}; \left(\frac{B_0^2}{4\pi Q - B_0^2}\right)^{1/2}; 0; 0\right)$$

the Lorentz-factor

$$u^0 = \left(\frac{4\pi Q}{4\pi Q - B_0^2}\right)^{1/2}$$

the 3D velocity

$$v_x = \frac{cB_0}{\sqrt{4\pi Q}} = \frac{cB_0}{\sqrt{4\pi\rho_0\left(c^2 + \frac{1}{\gamma-1} c_s^2 + v_a^2\right)}} \equiv U_A$$

the magnetic field

$$h^k = \left(0;\ 0;\ 0;\ \left(\frac{4\pi Q - B_0^2}{B_0^2}\right)^{1/2} E^*\left(x^0 - \frac{c}{U_A} x^1\right)\right)$$

the shock wave shape

$$f\left(x^0, x^1\right) = \frac{x^1 \rho_0}{u^1 \rho B_0} E^*\left(x^0 - \frac{c}{U_A} x^1\right)$$

where $c_s^2 = \gamma \frac{p_0}{\rho_0}$ $\quad v_a^2 = \frac{B_0^2}{4\pi\rho_0}$ $\quad Q = \rho_0\left(c^2 + \frac{1}{\gamma-1} c_s^2 + v_a^2\right)$

The reconnection is especially efficient for the case of strong magnetic field $\frac{B_0^2}{8\pi} >> \rho_0 c^2$ and cold plasma $p_0 << \frac{B_0^2}{8\pi}$

The Lorentz-factor turns out to be:

$$u^0 = \left(\frac{v_a^2}{c^2 + c_s^2/(\gamma-1)}\right)^{1/2}$$

plasma is accelerated up to relativistic velocities

$$v_x = c - \frac{c}{2v_a^2}\left(c^2 + \frac{c_s^2}{\gamma-1}\right)$$

and simultaneously significantly is compressed and is heated

$$\rho = \frac{\rho_0}{\gamma-1} \frac{\gamma v_a/2}{\sqrt{c^2 + c_s^2/(\gamma-1)}} \qquad T^{00} = \frac{\rho_0}{\gamma-1} \frac{\gamma v_a^4/2}{c^2 + c_s^2/(\gamma-1)}$$

Plasma density ρ and energy density $T^{00} \to \infty$ when $\frac{B_0^2}{8\pi} >> \rho_0 c^2$ $\quad \frac{B_0^2}{8\pi} >> p_0$

At the same time the z-size of the outflow region $z_{or} \sim (c^2 + c_s^2/(\gamma-1))/v_a^2 \to 0$. Hence, z_{OR} becomes much less than the z-size of the reconnected magnetic flux tube. This means that the relativistic plasma is concentrated to a small volume.

Analysis of the solution

- Reconnected magnetic flux and energy inside OR-region are completely defined by the behavior of electric field along the reconnection line
- The plasma is accelerated up to relativistic Alfvén velocity independently of electric field behavior along the reconnection line
- Plasma is highly compressed and heated at the slow shocks.
- The magnetic energy is transformed into plasma energy.

The inflow region

1. Linearization $\quad \partial_k T^{ik} = 0$
2. Displacement 4-vector ξ^i:

$$\delta\rho = -\partial_i\left(\rho\xi^i\right) + \rho u_l u^j \partial_j \xi^l$$
$$\delta u^i = u^l \partial_l \xi^i - \xi^l \partial_l u^i - u^i u_l u^j \partial_j \xi^l$$
$$\delta h^i = h^l \partial_l \xi^i - \xi^l \partial_l h^i - h^i \partial_l \xi^l + h^i u_l u^j \partial_j \xi^l$$

3. Transformation into the Fourier-Laplace space $\quad \partial_0 \to p, \partial_1 \to ik$
4. Equation for the normal component of the displacement vector ξ^3:

$$\partial_{33}\xi^3 - a^2\xi^3 = 0$$

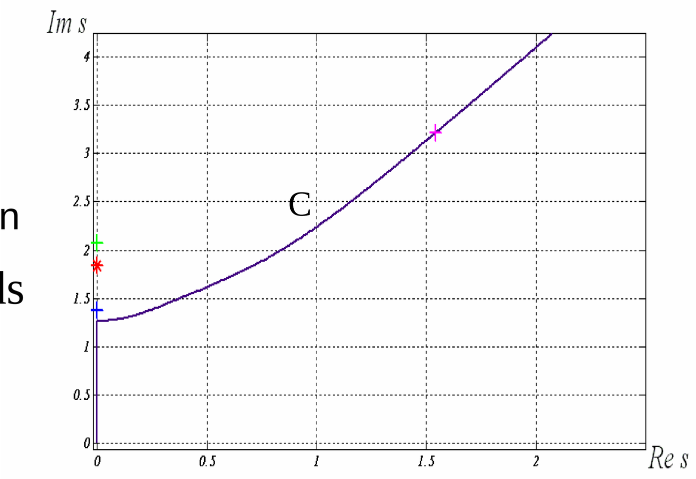
with

$$a^2 = \frac{\left(wp^2 + \left(p^2 + k^2\right)v_a^2\right)\left(wp^2 + c_s^2 k^2\right)}{\left(c_s^2 + v_a^2\right)wp^2 + v_a^2 c_s^2 k^2}$$

The disturbance of the plasma density in the inflow region

$$\delta\rho = \frac{1}{\pi}\mathrm{Re}\int_C K(s)E(t - \tau(s))ds$$

C – Cagniard contour, $\quad s = k/p$

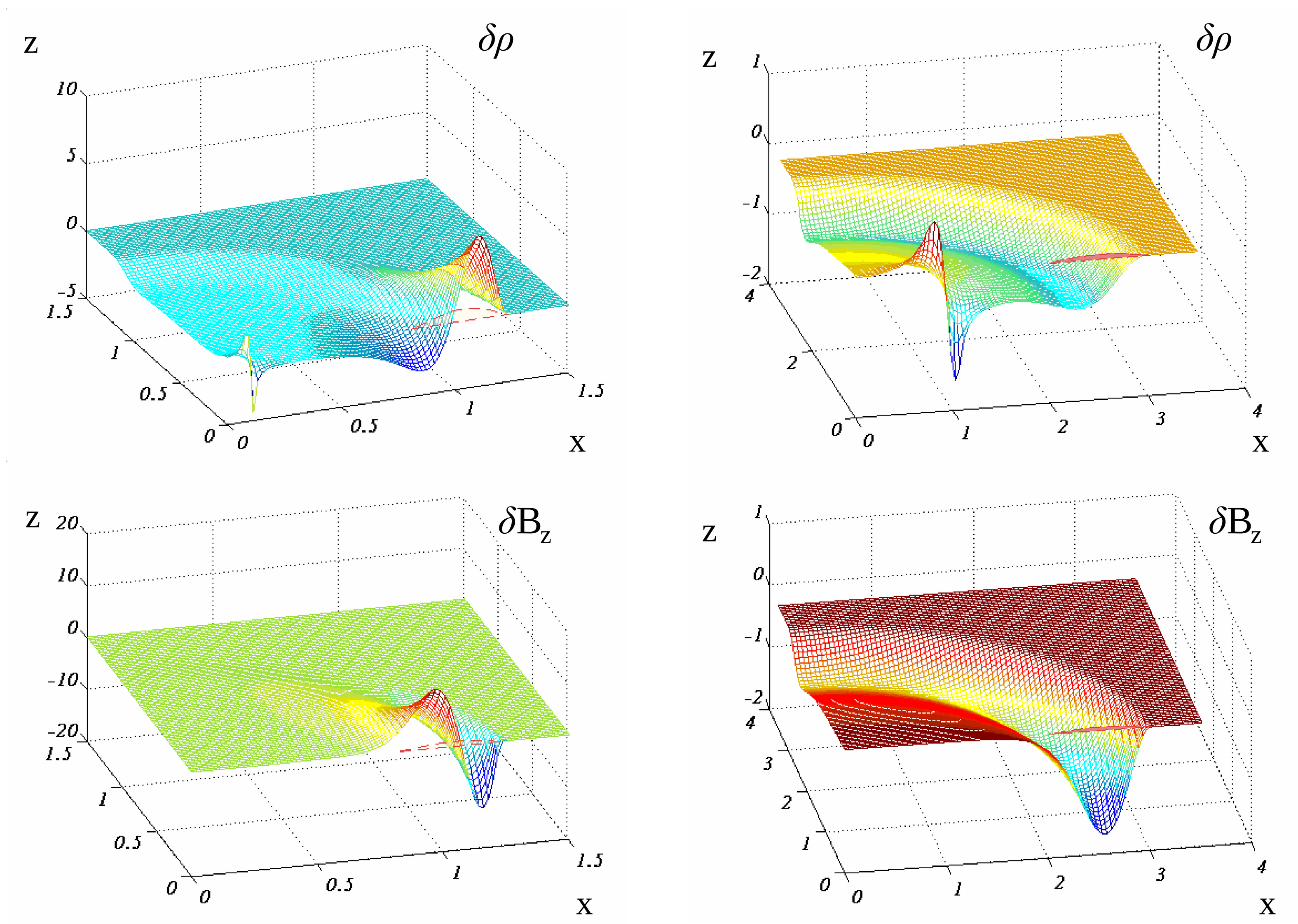


Non-relativistic

$\frac{8\pi\rho_0 c^2}{B_0^2} \sim 10$ $\quad \frac{8\pi p_0}{B_0^2} \sim 0.01$

Ultra-relativistic

$\frac{8\pi\rho_0 c^2}{B_0^2} \sim 10^{-3}$ $\quad \frac{8\pi p_0}{B_0^2} \sim 10^{-5}$

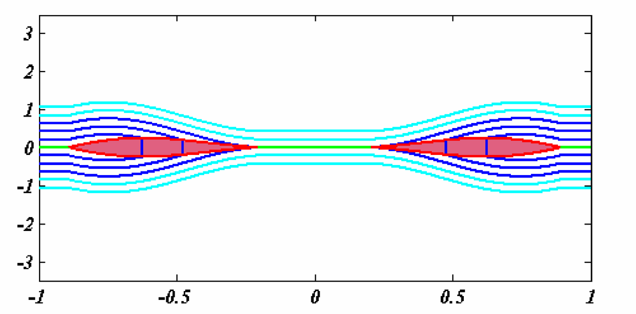


Comparison of non-relativistic and relativistic reconnection

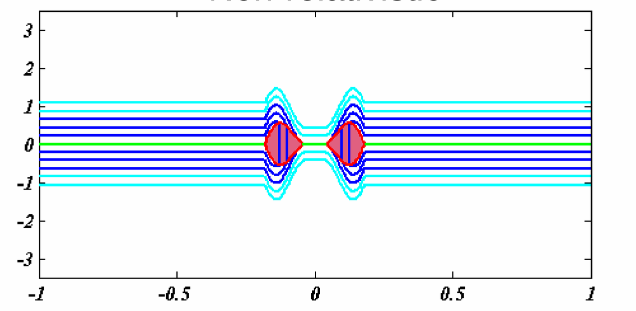
	Non-relativistic	Relativistic
Compression	$\rho = \frac{\gamma}{\gamma-1} \approx 2.5$	$\rho \to \infty$
Acceleration	$v_a = \frac{B_0}{\sqrt{4\pi\rho_0}}$	$U_a \to c$
Efficiency	$\eta \approx \frac{1}{2}$	$\eta \to 1$

The efficiency is defined by the ratio of the total energy within the outflow region to the reduction of magnetic energy in the whole space.

Relativistic



Non-relativistic



Conclusions

- The reconnection is especially efficient for the case of strong magnetic field and cold plasma.
- The plasma is accelerated to the nearly speed of light, and the magnetic energy is effectively converted to the plasma energy with very small characteristic time.
- The plasma and the energy are concentrated into a very tiny volume so that the size of the outflow region in the normal z direction tends to zero. Therefore disturbances produced by the outflow region in the surrounding media are also small compared with those generated by the reconnection flux.

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