Physical aspects of the BZ-process

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Plan of this talk

- ***3+1 formulation of Black Hole Electrodynamics***:
  Macdonald & Thorne versus ***Landau*** and ***Komissarov*** ;
- ***Magnetohydrodynamics (MHD)*** versus ***Magnetodynamics (MD)***;
- ***Horizon*** versus ***Ergosphere***;
- ***Blandford-Znajek*** versus ***Punsly-Coroniti***;
- ***Kerr-Schild*** coordinates versus ***Boyer-Lindquist*** coordinates;
- ***Znajek's "boundary condition***" versus ***regularity condition***;
- Insights from ***Numerical Simulations***;

3+1 Electrodynamics of Black Holes

4-tensor formulation: $\nabla_\nu {}^*F^{\mu\nu} = 0, \quad \nabla_\nu F^{\mu\nu} = I^\mu$

${}^*F^{\mu\nu}$ - Faraday, $F^{\mu\nu}$ - Maxwell, I^μ - 4-current

3+1 splitting:

${}^*F^{\mu\nu} \to (\vec{B}, \vec{E}), \quad F^{\mu\nu} \to (\vec{H}, \vec{D}), \quad I^\mu \to (\vec{J}, \rho)$

$x^\nu \to (x^i, t)$

$g_{\mu\nu} \to (\alpha, \vec{\beta}, \gamma_{ij})$

γ_{ij}- metric 3-tensor of space (t=const)

α- lapse function,

$\vec{\beta}$- shift vector

FIDO – fiducial observer; it is at rest in the space

3+1 Electrodynamics of Black Holes

Macdonald & Thorne (1980):

$$\partial_t \vec{\vec{B}} - \mathcal{L}_{\vec{\beta}} \vec{\vec{B}} + \vec{\nabla} \times \alpha \vec{\vec{E}} = 0$$

$$-\partial_t \vec{\vec{E}} + \mathcal{L}_{\vec{\beta}} \vec{\vec{E}} + \vec{\nabla} \times \alpha \vec{\vec{B}} = \alpha \vec{j}$$

$$\vec{\nabla} \cdot \vec{\vec{E}} = \rho \qquad \vec{\nabla} \cdot \vec{\vec{B}} = 0$$

$\vec{\vec{E}}, \vec{\vec{B}}, \vec{j}, \rho$ are as seen by FIDOs

Komissarov (2004) (**Landau** 1951):

$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$-\partial_t \vec{D} + \vec{\nabla} \times \vec{H} = \vec{J}$$

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{E} = \alpha \vec{D} + \vec{\beta} \times \vec{B}$$

$$\vec{H} = \alpha \vec{B} - \vec{\beta} \times \vec{D}$$

$$\vec{\vec{E}} = \vec{D}, \quad \vec{\vec{B}} = \vec{B}, \quad \vec{J} = \alpha \vec{j} - \rho \vec{\beta}$$

3+1 Electrodynamics of Black Holes

Macdonald & Thorne (1980):

- advantages: 1) clear physical meaning of involved parameters;
- disadvantages: 1) more complicated equations;
  2) holds only for foliations with $\vec{\nabla}\cdot\vec{\beta}=0$,
  hence not for Kerr-Schild foliation;

Komissarov (2004):

- advantages: 1) very familiar and simple equations;
  2) holds for any foliation where $g_{\mu\nu,t}=0$;
  3) clearly shows that the space around BH behaves
  as an **electromagnetically active medium**;
- disadvantages: cannot see any.

Properties of stationary axisymmetric vacuum solutions

a)

stationarity axisymmetry vacuum

$\longrightarrow$ $E_\phi = D_\phi = H_\phi = B_\phi = 0$

purely poloidal fields !

b) If $\vec{B} \neq 0$ then $\vec{D} \neq 0$. Thus, **FIDOs always see some electric field around BH !**

Proof (by contradiction): Assume that $\vec{D} = 0$. Then

$$\vec{E} = \alpha\vec{D} + \vec{\beta}\times\vec{B} \quad \rightarrow \quad \vec{E} = \vec{\beta}\times\vec{B}$$

$$\vec{H} = \alpha\vec{B} - \vec{\beta}\times\vec{D} \quad \rightarrow \quad \vec{H} = \alpha\vec{B}$$

$$\partial_t\vec{B} + \vec{\nabla}\times\vec{E} = 0 \quad \rightarrow \quad \vec{\nabla}\times(\vec{\beta}\times\vec{B}) = 0$$

$$-\partial_t\vec{D} + \vec{\nabla}\times\vec{H} = \vec{J} \quad \rightarrow \quad \vec{\nabla}\times(\alpha\vec{B}) = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

overdetermined system; no nontrivial solutions for B

QED

Properties of stationary axisymmetric vacuum solutions

c) ***Inside the BH ergosphere this electric field has unscreened component capable of accelerating charged particles and driving electric currents!*** That is one cannot have simultaneously

$$\vec{D}\cdot\vec{B}=0 \quad \text{and} \quad B^2-D^2>0 \ .$$

Proof: Suppose that $\vec{D}\cdot\vec{B}=0$. Then $\vec{E}\cdot\vec{B}=0$ and $\vec{E}=-\vec{\omega}\times\vec{B}$, where $\vec{\omega}=\Omega\vec{i}_\phi$ and $\Omega=\mathrm{const}$ along B.

If E is not created by external charges then E=0 at infinity, hence Ω=0 at infinity, hence E=0 everywhere. Then

$$\vec{D}=-\frac{1}{\alpha}\vec{\beta}\times\vec{B} \quad \rightarrow \quad B^2-D^2=\frac{\alpha^2-\beta^2}{\alpha^2}B^2$$

Inside the ergosphere $\alpha^2-\beta^2<0$ and thus $B^2-D^2<0$

QED

Example: vacuum solution by Wald (1974).

Properties of stationary axisymmetric vacuum solutions

d) ***Inside the ergosphere the electric field cannot be screened by any static distribution of electric charge*.**

Proof: With charges present $H_\phi = I_{pol}/2\pi$ may not vanish.

Now assuming $\vec{D} \cdot \vec{B} = 0$ we obtain

$$\alpha^2(B^2 - D^2) = B^2(\alpha^2 - \beta^2) + \left(\frac{\beta^\phi}{\alpha}\right)^2 H_\phi^2$$

Inside the ergosphere $B^2 - D^2 > 0$ only if $I_{pol} \neq 0$!

QED

When plenty of free charges are supplied into the magnetosphere (e^+-e^- pairs) they are forced to keep moving (electric current) by the marginally screened electric field.

Electromagnetic extraction of energy from Kerr BHs

Steady-state force-free magnetospheres:

$$\vec{\nabla}\times\vec{E} = 0$$
$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{H} = \vec{J}$$
$$\vec{\nabla}\cdot\vec{D} = \rho$$
$$\rho\vec{E} + \vec{J}\times\vec{B} = 0$$

$$\vec{E} = -\vec{\omega}\times\vec{B}, \ \ \vec{w} = \Omega\vec{i}_\phi$$

Ω and $H_\phi = I_{pol}/2\pi$ are constant along magnetic field lines

Angular momentum flux:

$$\vec{L}_{pol} = -H_\phi \vec{B}_{pol} \quad (\vec{\nabla}\cdot\vec{L} = 0)$$

Energy flux:

$$\vec{S}_{pol} = -\Omega H_\phi \vec{B}_{pol} \quad (\vec{\nabla}\cdot\vec{S} = 0)$$

Electromagnetic extraction of energy from Kerr BHs

Blandford and Znajek (1977)

- Steady-state force-free equations;
- Monopole magnetic field;
- Slowly rotating BH, $a \ll 1$;
- Boyer-Lindquist coordinates;
- Znajek's horizon boundary conditions.

$$\Omega = a/8$$

$$H_\phi = -\Omega B_0 \sin^2 \theta$$

Punsly and Coroniti (1990)

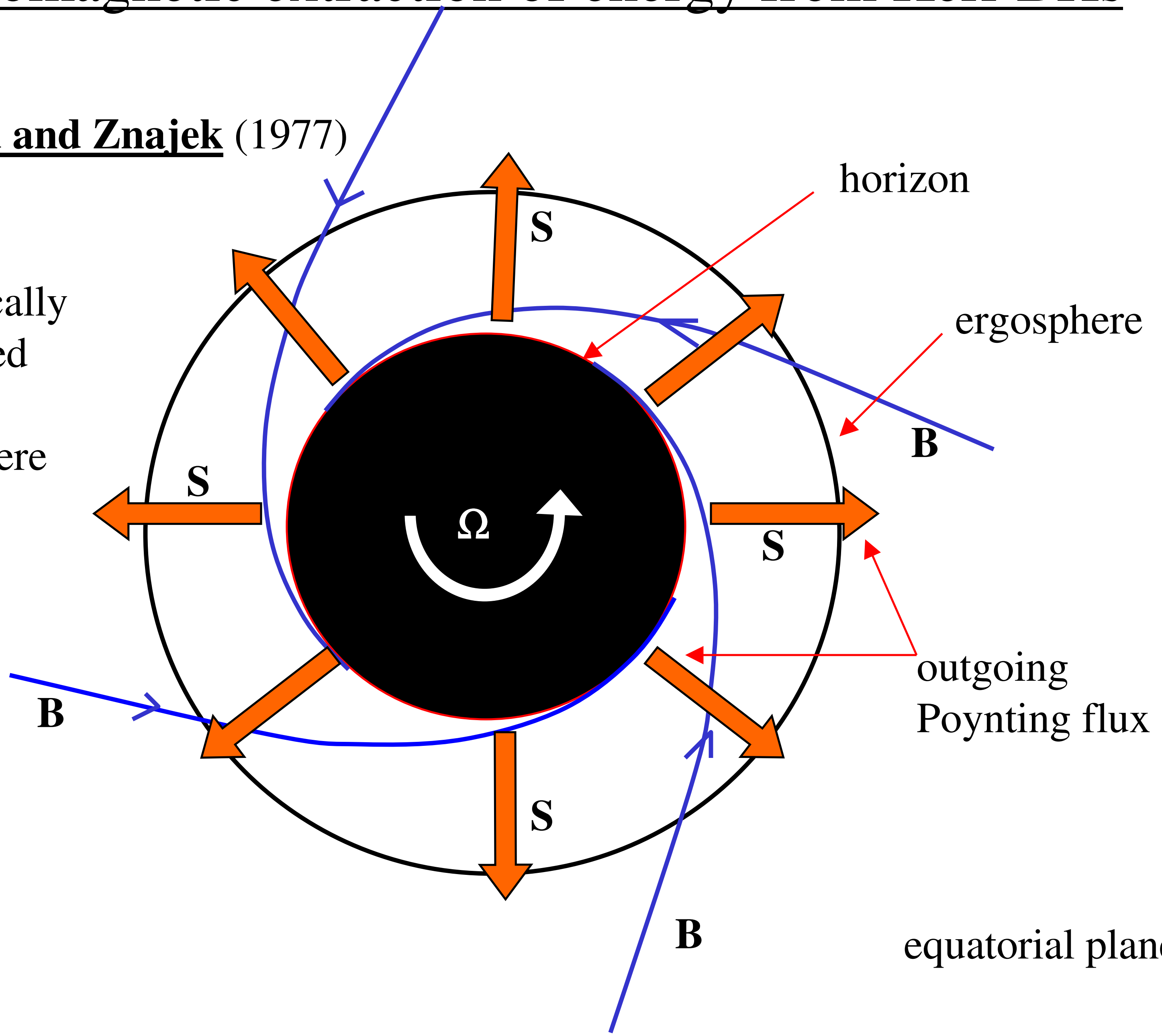
- Horizon is causally disconnected! One cannot set boundary conditions on the horizon; event horizon is not a unipolar inductor;
- BZ-solution must be unstable;
- Force-free approximation breaks down near the horizon;
- Particle inertia plays a key role. Only MHD will do. "MHD Penrose process"

Macdonald &Thorne (1982): Horizon acts like a unipolar inductor

Electromagnetic extraction of energy from Kerr BHs

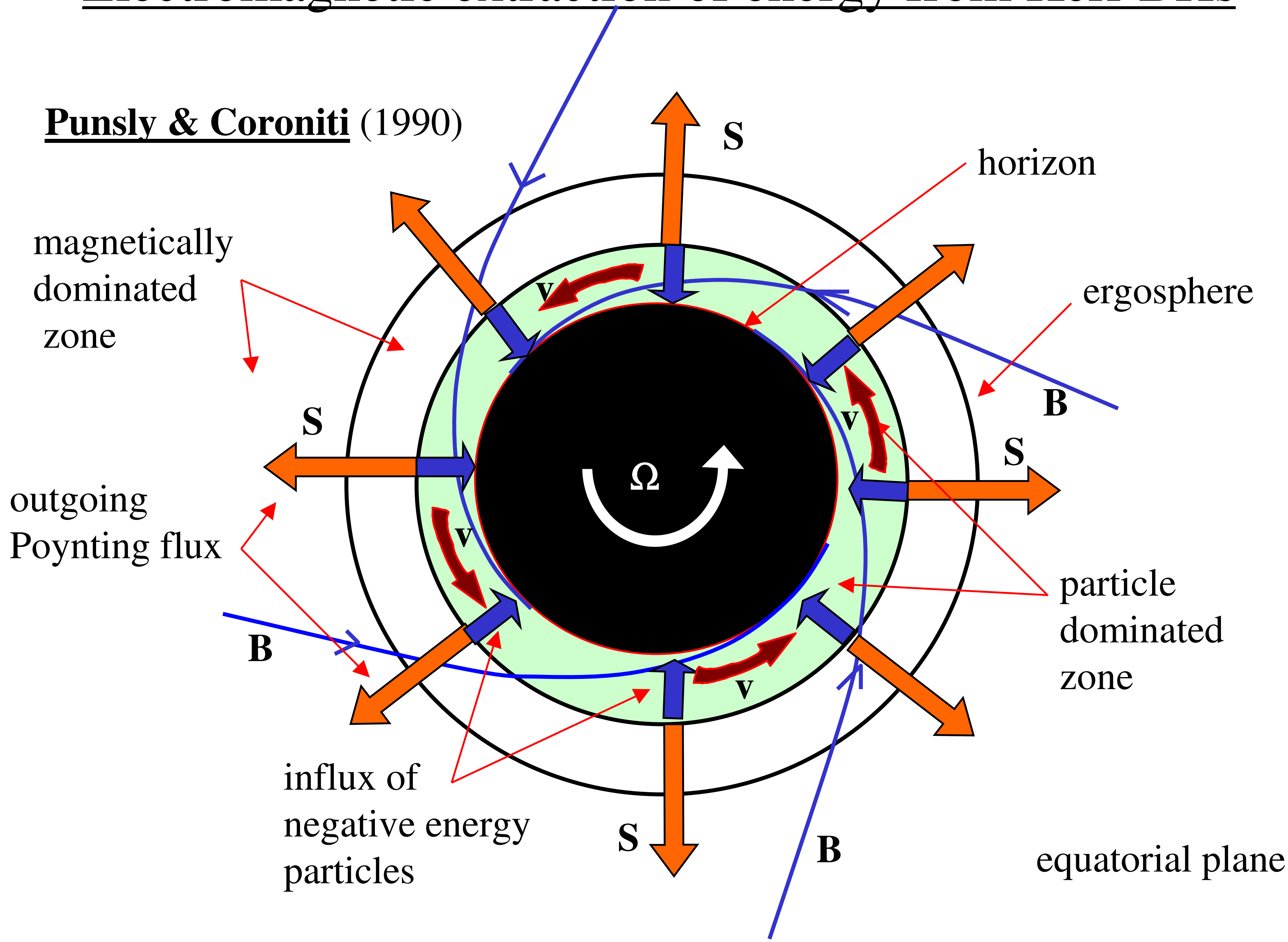
Blandford and Znajek (1977)

magnetically
dominated
plasma
everywhere



Electromagnetic extraction of energy from Kerr BHs

Punsly & Coroniti (1990)



Magnetohydrodynamics versus *Magnetodynamics*

Ideal relativistic ***magnetohydrodynamics (MHD)***

$$\nabla_\alpha \rho u^\alpha = 0$$ - continuity equation

$$\nabla_\alpha {}^*F^{\alpha\beta} = 0$$ - Faraday equation

$$\nabla_\alpha T^{\alpha\beta} = 0$$ - energy-momentum equation

$$F_{\nu\mu} u^\mu = 0$$ - perfect conductivity condition

$$T^{\alpha\beta} = T_{\mathsf{f}}^{\alpha\beta} + T_{\mathsf{m}}^{\alpha\beta}$$ - total stress-energy-momentum tensor

$$T_{\mathsf{f}}^{\alpha\beta} = F^\alpha{}_\mu F^{\beta\mu} - \frac{1}{4}(F_{\mu\nu}F^{\mu\nu})g^{\alpha\beta}$$ - electromagnetic field contribution

$$T_{\mathsf{m}}^{\alpha\beta} = w u^\alpha u^\beta + p g^{\alpha\beta}$$ - particle contribution

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$$\nabla_\alpha T^{\alpha\beta} = 0$$ - energy-momentum equation

$$\vec{\breve{E}} \cdot \vec{\breve{B}} = 0, \quad \breve{B}^2 - \breve{E}^2 > 0$$ - perfect conductivity conditions

$$T^{\alpha\beta} = T_{\mathsf{f}}^{\alpha\beta}$$ - total stress-energy-momentum tensor

$$T_{\mathsf{f}}^{\alpha\beta} = F^\alpha_{\ \mu} F^{\beta\mu} - \frac{1}{4}(F_{\mu\nu}F^{\mu\nu})g^{\alpha\beta}$$ - electromagnetic field contribution

Magnetohydrodynamics versus *Magnetodynamics*

Ideal relativistic ***magnetodynamics (MD)***

$$\nabla_\alpha {}^*F^{\alpha\beta} = 0$$ - Faraday equation

$$\nabla_\alpha T^{\alpha\beta} = 0$$ - energy-momentum equation

$${}^*F_{\mu\nu}F^{\mu\nu} = 0, \quad F_{\mu\nu}F^{\mu\nu} > 0$$ - perfect conductivity conditions

$$T^{\alpha\beta} = F^{\alpha}_{\ \mu}F^{\beta\mu} - \frac{1}{4}(F_{\mu\nu}F^{\mu\nu})g^{\alpha\beta}$$

Magnetohydrodynamics versus *Magnetodynamics*

Properties *of* ***Magnetodynamics (MD)***

- This is a hyperbolic system of conservation laws (Komissarov 2002);
- It has two hyperbolic waves, *fast* and *Alfven*. Both propagate with the speed of light;
- Magnetic field vanishes in the "fluid frame", that is frame moving with the local drift velocity;
- It describes flow of magnetic mass-energy under the action of magnetic pressure and tension;
- It has alternative formulations (Uchida 1997, Gruzinov 1999);
- One can add resistivity (Lyutikov 2003, Komissarov 2004 Komissarov et al., 2006);
- It has an alternative name , *Force Free Degenerate Electrodynamics,* but *Magnetodynamics* is a better name !;

Kerr-Schild versus Boyer-Lindquist coordinates

Boyer-Lindquist coordinates $\{t,r,\theta,\phi\}$

- Stationary axisymmetric metric form that becomes Minkowskian at infinity;
- There exists a coordinate singularity on the event horizon $(r = r_+)$;
- The hyper-surface t=const is space-like outside of the event horizon, null on the event horizon, and time-like inside it (Horizon is "the end of space");
- FIDOs are proper observers outside of the event horizon, "*luminal*" on the event horizon, and "*superluminal*" inside of it;

Quantities that are defined in 3+1 formulations as seen by FIDOs become meaningless in the limit $r \rightarrow r_+$ ***!***

This singularity is a source of many confusions … .

Kerr-Schild versus Boyer-Lindquist coordinates

Kerr-Schild coordinates $\{t,r,\theta,\phi\}$

- Stationary axisymmetric metric form that becomes Minkowskian at infinity;
- There is no coordinate singularity on the event horizon $(r = r_+)$
- The hyper-surface t=const is space-like for any r (Horizon is not "the end of space");
- FIDOs are proper observers for any r;

Quantities that are defined in 3+1 formulations as seen by FIDOs always make sense!

- In Boyer-Lindquist coordinates Kerr-Schild FIDOs move radially towards the space-time singularity at r=0;
- Metric form has 2 more non-vanishing terms compared to the Boyer-Lindquist one.

Znajek's boundary condition versus regularity condition

In Magnetodynamics the fast speed=c;
In Kerr-Schild coordinates the horizon is inside space;

→ The horizon is a critical surface !

Following Weber&Davis (1967) we can relate B^{ϕ} and B^{r} :

$$B^{\phi} = \frac{\alpha H_{\phi} - B^{r} \sin^{2}\theta (2r\Omega - a)}{\Delta \sin^{2}\theta}$$

$\Delta=0$ on the event horizon, $r=r_{+}$, and to keep B^{ϕ} finite we need

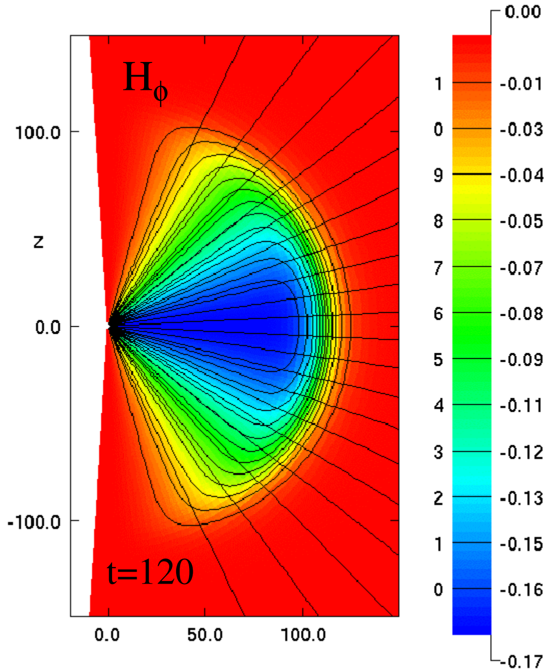
$$\alpha_{+} H_{\phi} - B^{r} \sin^{2}\theta (2r_{+}\Omega - a) = 0$$

-- Znajek's "boundary condition"

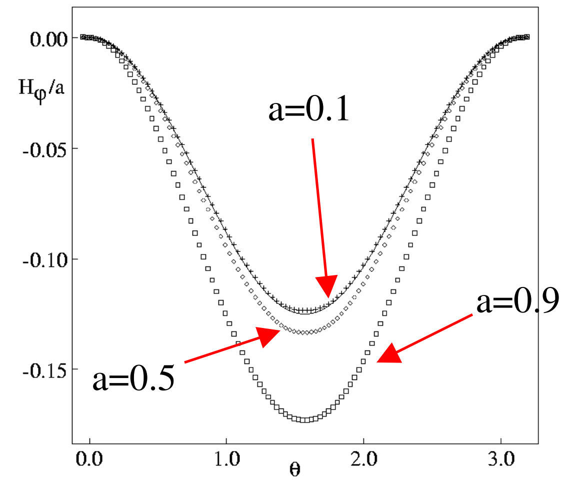
Znajek's condition is a regularity condition!

Numerical Simulations

Monopole field; MD simulations (Komissarov 2001, McKinney 2005)



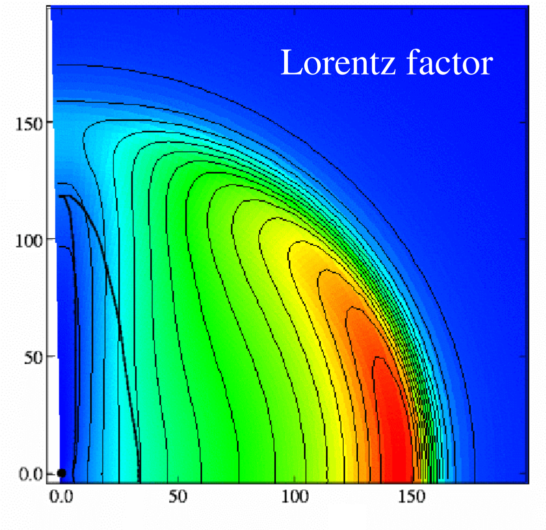
•Kerr-Schild coordinates;
•Inner boundary is inside the event horizon;
•Initially non-rotating monopole field;



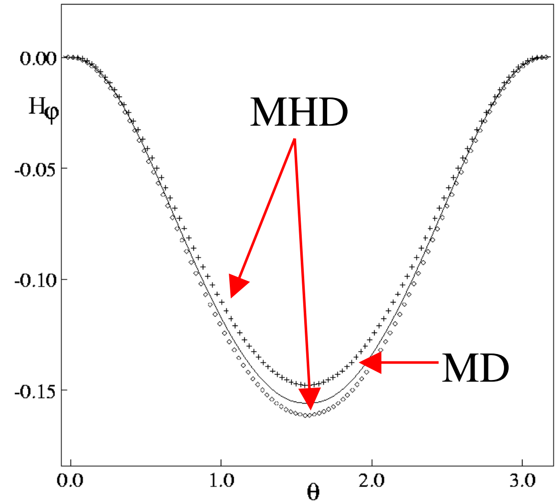
1) Numerical solution relaxes to the steady-state analytic solution of Blandford & Znajek(1977) !
(The stability issue is closed.)
2) No indications of a singular behaviour at the event horizon. (May be in MHD?)

Numerical Simulations

Monopole field; MHD simulations (Komissarov 2004)



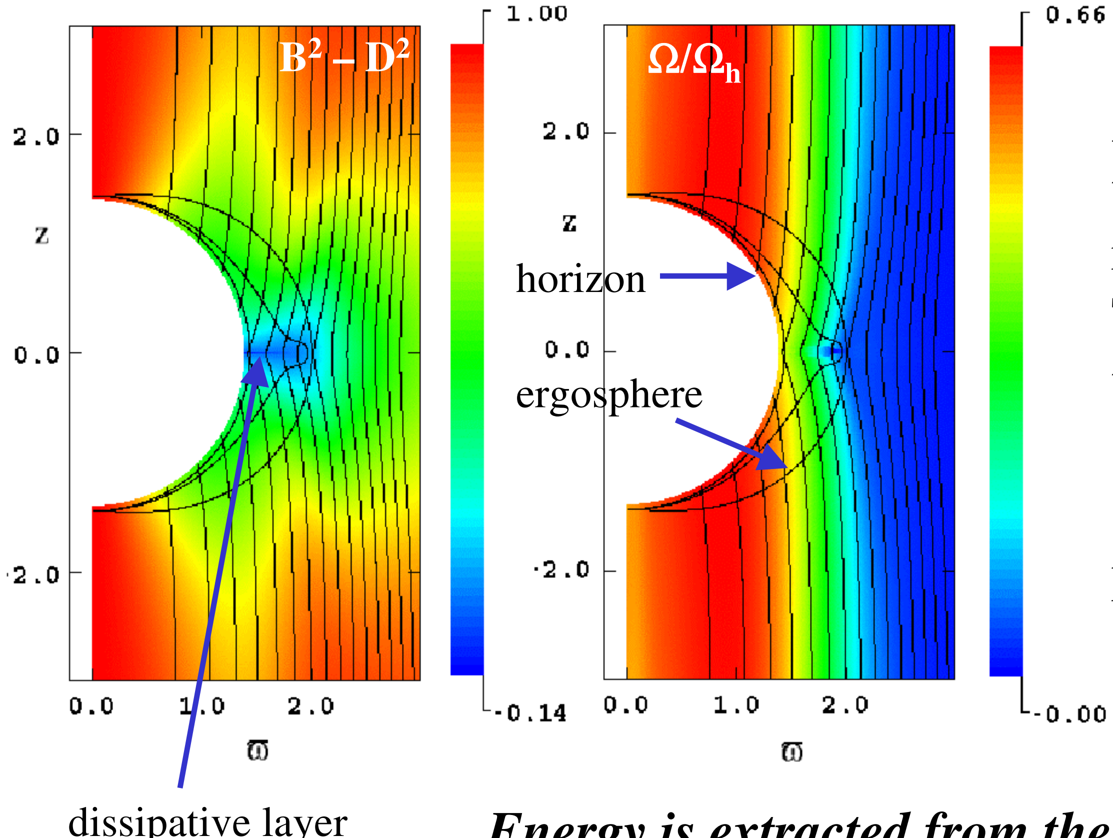
- Kerr-Schild coordinates;
- Inner boundary is inside the event horizon;
- Initially non-rotating monopole field;
- Initially plasma is at rest relative to FIDOs;



1) Magnetically dominated MHD solution is close to the MD solution;
2) No indications of a singular behaviour at the event horizon.
Hence ***no break down of MD approximation at the event horizon*** *contrary to Macdonald & Thorne(1982), Punsly & Coroniti(1990), Lee(2006).*

Numerical Simulations

Uniform field; MD simulations (Komissarov 2004)

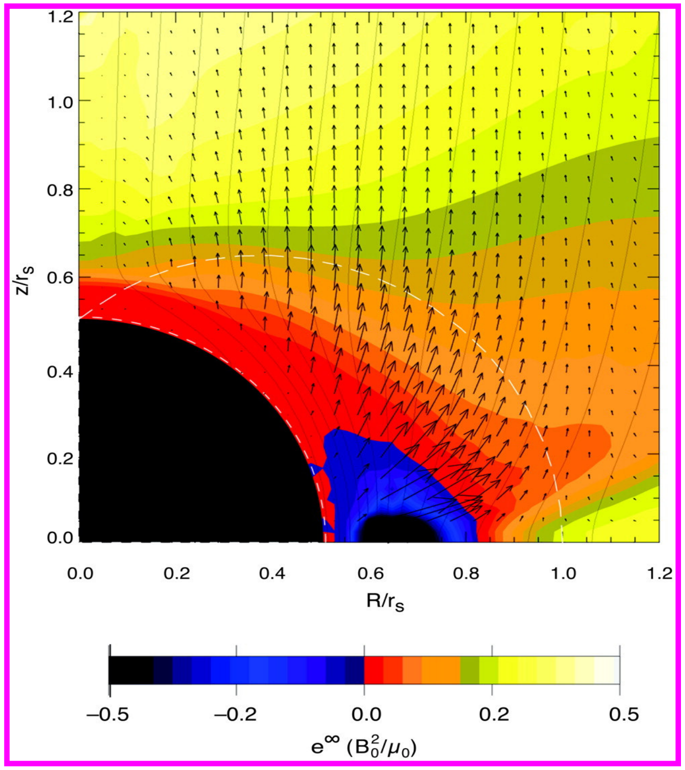


All field lines which enter the ergosphere are set in rotation.
The dissipative layer in the equatorial plane acts as an energy source.
(It emits negative energy photons that fall into BH)

Energy is extracted from the space between the horizon and the ergosphere! Just like in Penrose mechanism.

Numerical Simulations

Uniform field; MHD simulations (Koide et al. 2002,2003)



Solution at $t \sim 14\ r_g/c$ (~ one period of the black hole)

•Region of *negative mechanical energy* develops within the *ergosphere*;

•Near the horizon the outgoing Poynting flux is of the same order as the outgoing mechanical energy flux;

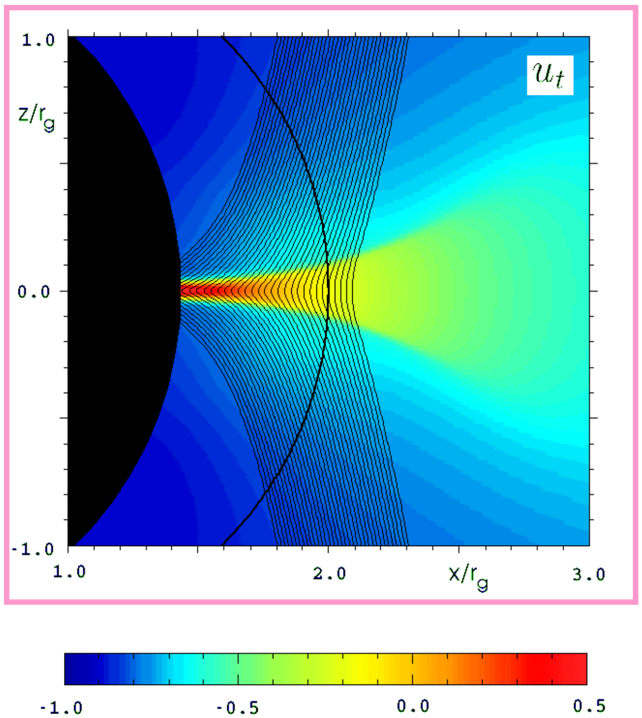
•This partly agrees with the model by Punsly & Coroniti (MHD Penrose process!)

However, a steady state is not reached! Could this be only a transient phase?

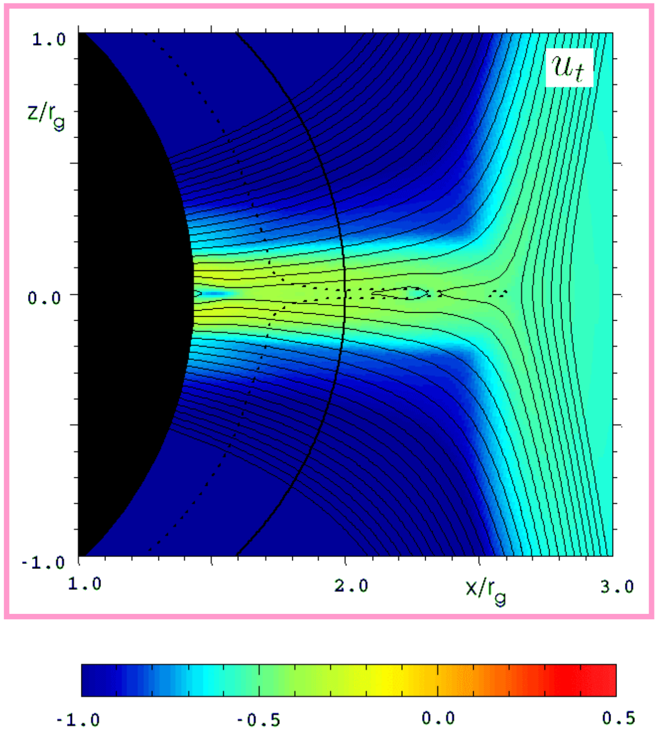
Numerical Simulations

Uniform field; MHD simulations (Komissarov, 2005)

$t = 6\ r_g/c$



$t = 60\ r_g/c$



The solution settles to a steady state with a split-monopole configuration where only the Blandford-Znajek process operates.

Numerical Simulations

BZ-process in BH-accretion disc problems; MHD simulations

- **Koide et al**. (1999): BL-coordinates, thin disk, short run, transient ejection from the disk (?);
- **Komissarov** (2001): BL-coordinates; wind from the disc; outflow in magnetically-dominated funnel (BZ-process?);
- **McKinney&Gammie** (2004), **McKinney** (2005): KS-coordinates, outflow in magnetically-dominated funnel – clear indications of the BZ-process;
- **Hirose et al**. (2004 - 2006): BL-coordinates; outflow in the funnel (BZ-process?);
  but **Punsly** (2006): MHD-Penrose process or computational errors?

Conclusions

1. The Blandford-Znajek process has its roots in the electromagnetic properties of curved space-time of BHs. The space around them is an ***electromagnetically active medium*** (new 3+1 formulation of Black Hole Electrodynamics; E,B,H,D-fields).

2. ***The event horizon has no active role*** to play in the BZ-process (apart from a superficial one that is given to it in The Membrane Paradigm). Like in the mechanical Penrose process ***the key surface is the ergosphere***. Marginal screening of electric field (D) in pair-filled ergosphere is accomplished by means of poloidal electric currents.

3. The undue emphasis on the event horizon is caused by the coordinate singularity of the widely used ***Boyer-Lindquist coordinates*** where it appears as "the end of space". The ***Kerr-Schild coordinates*** remove this confusion.

Conclusions

4. Structure and dynamics of magnetically dominated pair-plasma magnetospheres of BHs is well described within the approximation of ***Magnetodynamics (MD).*** The BZ-solution is a steady-state MD-solution.

5. There are ***no causality problems*** associated with BZ-process. The so-called "horizon boundary condition" of Znajek is simply a ***regularity condition***. In the MD-limit the event horizon coincides with the ***fast critical surface*** (of the ingoing wind).

6. Contrary to the *theory of Punsly-Coroniti* ***particle-dominated regions do not spontaneously develop*** at "the base" (event horizon) of magnetically-dominated BH magnetospheres. MD approximation remains valid across the event horizon. ***The MHD Penrose mechanism*** is unlikely to play a significant role in powering Poynting-dominated outflows from BHs.