TURBULENCE AND PARTICLE ACCELERATION

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OUTLINE

I. PARTICLE ACCELERATION: General

II. STOCHASTIC ACCELERATION BY TURBULENCE

III. SOME APPLICATIONS

I. ACCELERATION: General

A. **Observed over a wide energy, time and spatial scales**

B. **Cosmic Rays (electrons, protons and ions; isotopes)**

C. **Radiating Sources (electrons and protons)**

Some General Requirements

- Acceleration of Background Particles (no pre-acceleration)
- Losses at the Acceleration Site (Coulomb, Synchrotron, Compton)

Astrophysical Sources
Magnetized Plasmas

1. Solar Flares
2. Sgr A* (slow accreting AGNs)
3. Accretion Disks
4. Clusters of Galaxies
5. AGN Jets
6. GRBs

ACCELERATION MECHANISMS

A: Electric Fields: **Parallel to B Field**

B: Fermi Acceleration

1. Shock or Flow Divergence: **First Order**

2. Stochastic Acceleration: **Second Order**

II. ACCELERATION MECHANISMS

A. ELECTRIC FIELDS: $\mathcal{E}$ (parallel to **B** field)

Acceleration Rate: $dp/dt = e\mathcal{E}$

Astrophysical Plasmas Highly Conductive: $\mathcal{E} \rightarrow 0$

Dricer Field: $\mathcal{E}_D = kT/(e\lambda_{\rm Coul})$

$\mathcal{E} < \mathcal{E}_D$: Energy Gain $\Delta E < kT(L/\lambda_{\rm Coul})$

$\mathcal{E} > \mathcal{E}_D$: Runaway Unstable Distribution Leads to

PLASMA TURBULENCE

1. Double Layers (DLs) in Earth's Magnetosphere

Multiple DLs: Difussive Process like

PLASMA TURBULENCE

2. Unipolar Induction in High B field of Neutron Stars
Extreme Relativistic Energies: Pair Cascade

II. ACCELERATION MECHANISMS

B. FERMI ACCELERATION

Random scattering by moving scattering centers.
Diffusive Process: Why Acceleration?
More headon than trailing scatterings
Phase space availibility

$$\frac{1}{p^2}\frac{\partial}{\partial p}(p^2 D_{pp}\frac{\partial f}{\partial p}) \rightarrow \frac{\partial}{\partial E}(D(E)\frac{\partial N}{\partial E}) - \frac{\partial}{\partial E}(A(E)N) \quad (1)$$

1. SHOCK ACCELERATION: (First Order Fermi)
Energy Gain: $\dot{p} = \frac{p}{3}\frac{\partial u}{\partial x}, \quad \delta p/p \sim U_{\rm shock}/v$
Need Scattering Agent *i.e.* **TURBULENCE**

Diffusive Shocks
Scattering Rate $D_{\rm scat}$
Acceleration Rate $\sim (U_{sh}/v)^2 D_{\rm scat}$

Relativistic Shocks
Most Energy Gained in First Passage
Most Likely in High B Plasmas *e.g.* GRBs or AGN Jets
Most of the Energy in Protons; How to convert to Electrons?

2. STOCHASTIC ACCELERATION:

(Second Order Fermi)

Plasma Waves or **TURBULENCE**

Energy Gain; *e.g.* Alfven Waves: $\delta p/p \sim (V_{\rm Alfven}/v)^2$

Scattering Rate $\sim D_{\rm scat}$

Acceleration Rate $\sim D_{pp}/p^2 \sim (V_{\rm Alfven}/v)^2 D_{\rm scat}$

•For $V_{\rm Alfven} > V_{\rm sound}$ TURBULENCE more efficient than SHOCKS

•At low energies or high B fields $D_{pp}/p^2 \gg D_{\rm scat}$ and TURBULENCE efficient accelerator

ACCELERATION MECHANISMS

A: Electric Fields: **Parallel to B Field**

Unstable leads to TURBULENCE

B: Fermi Acceleration

1. Shock or Flow Divergence: **First Order**

Shocks and Scaterers; i.e. TURBULENCE

2. Stochastic Acceleration: **Second Order**

Scattering and Acceleration by TURBULENCE

TURBULENCE

Shock Acceleration

- Simple model very attractive
- Some unanswered questions:

  Source Particles

  Scattering Processes

  Feedback and Nonlinear effects

Combined Turbulence and Shock Processes

II. STOCHASTIC ACCELERATION BY PLASMA TURBULENCE

1. Turbulence Generation
2. Turbulence Cascade
3. Turbulence Damping
4. Interactions with Particles
5. Spectrum of the Accelerated Particles

1. TURBULENCE GENERATION

Turbulence is Very Common in Astrophysics

Hydrodynamic: Ordinary Reynolds number

$$R_e = LV/\nu >>> 1; \quad \nu = \text{Viscosity}$$

In MHD: Magnetic Reynolds number

$$R_m = LV/\eta >>> 1; \quad \eta = \text{Mag. Diff. Coeff.}$$

Thus most flows or fluctuations lead to generation of turbulence on scales around L (or waves with k-vector $k_{min} = 1/L$)

2. TURBULENCE CASCADE

HD: Large eddies breaking into small ones

Eddy turnover or *cascade* time

$$\tau_{cas} \approx 1/kv(k) < L/V_{sound}$$

MHD: Nonlinear wave-wave interactions

$$\omega(k_1) = \omega(k_2) + \omega(k_3); \quad k_1 = k_2 + k_3$$

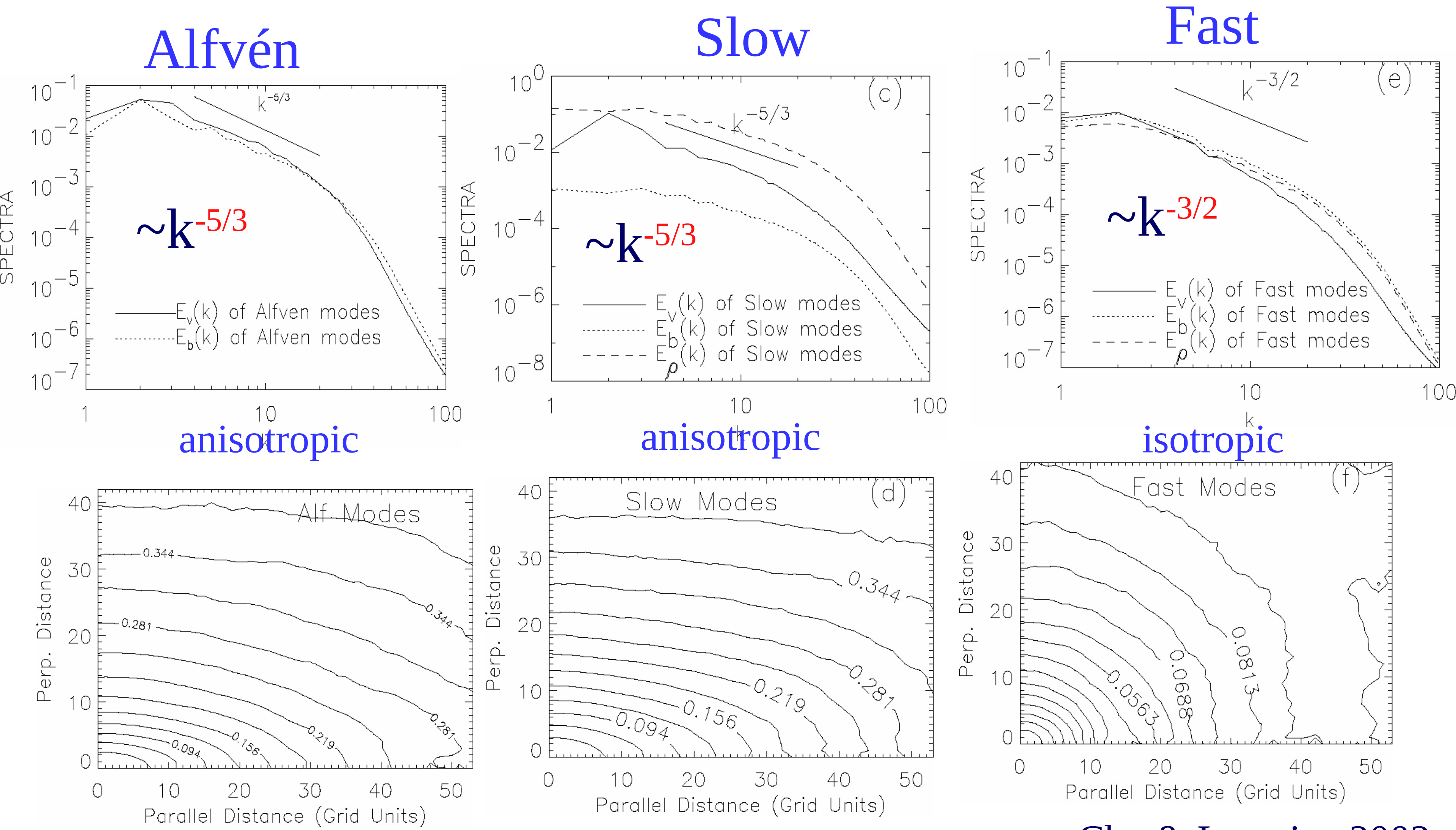
$$\tau_{cas} \le L/V_{Alfven}$$

Dispersion Relation: *(Low Beta Plasma,* $V_{Alfven} >> V_{Sound}$*)*

$$\omega(k) = k_{\|}V_{Alfven},\ kV_{Alfven},\ k_{\|}V_{Sound}$$

For Alfven, Fast and Slow Modes

2. Cascade of MHD Turbulence



Cho & Lazarian 2002

3. TURBULENCE DAMPING

Viscous or Collisional Damping: $l = k^{-1} >> \lambda_{Coul}$

Collisonless Damping: $k^{-1} << \lambda_{Coul}$

Thermal: *Heating of Plasma*

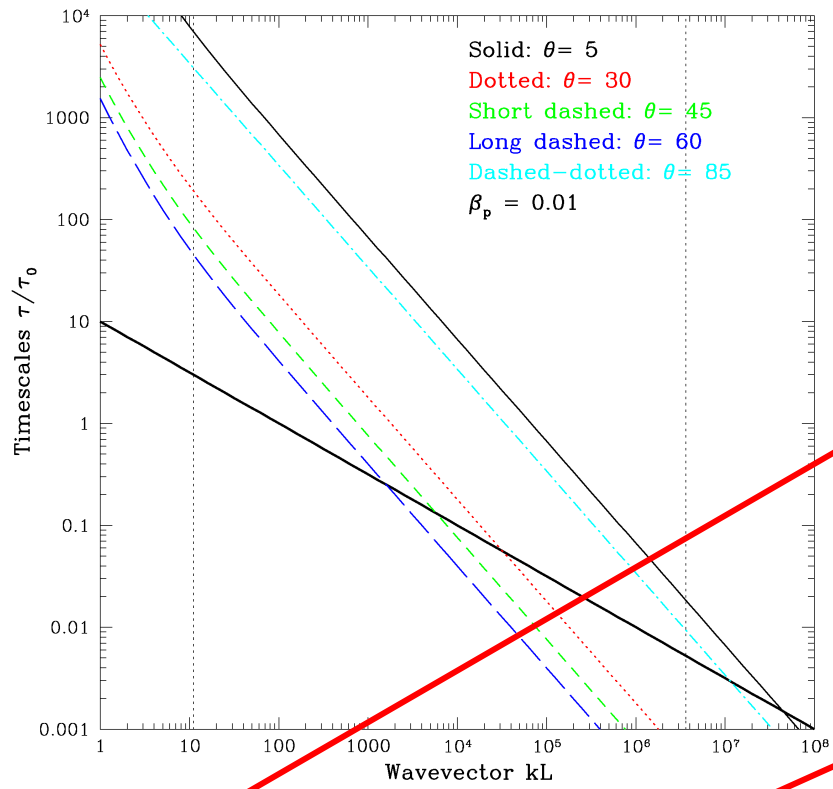
Nonthermal: *Particle Acceleration*

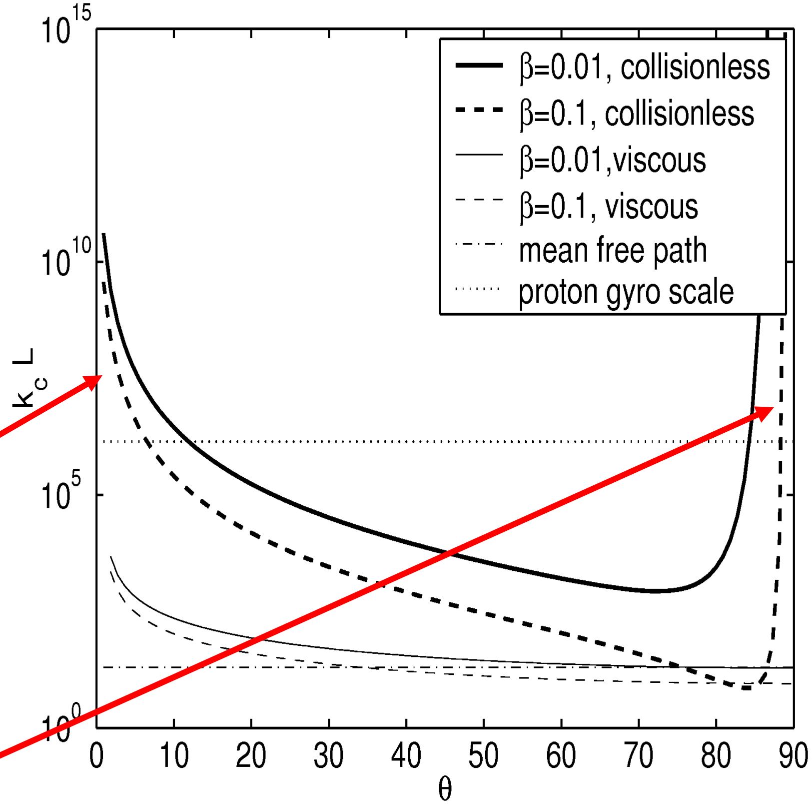
Turbulence is damped for $k > k_{\max}$

where $\tau_{damp}(\propto k^{-1}) = \tau_{cas}(\propto k^{-1/2})$

Inertial Range $k_{\min} < k < k_{\max}$

3. Turbulence Damping





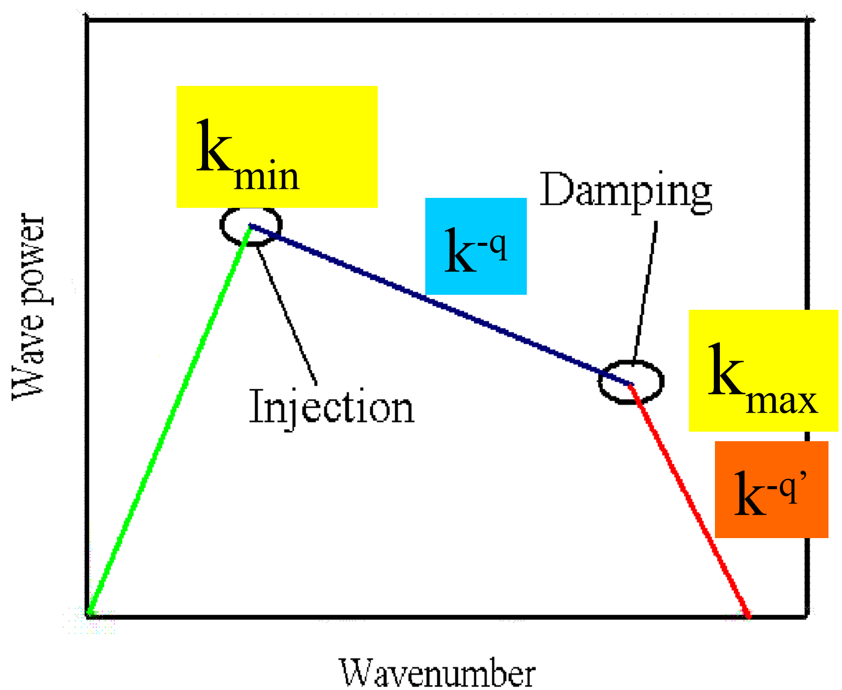
Parallel (and perpendicular) waves are not damped

Turbulence Spectrum

$W(k) = ?$

General Features:

- Injection scale: $k_{\min}$
- Cascade and index q
- Damping scale or $k_{\max}$



Kinetic Equation:

$$\frac{\partial W(\mathbf{k},t)}{\partial t} = \dot{Q}_p(\mathbf{k},t) - \gamma(\mathbf{k})W(\mathbf{k},t) + \nabla_i\left[D_{ij}\nabla_j W(\mathbf{k},t)\right] - \frac{W(\mathbf{k},t)}{T^W_{\rm esc}(\mathbf{k})}$$

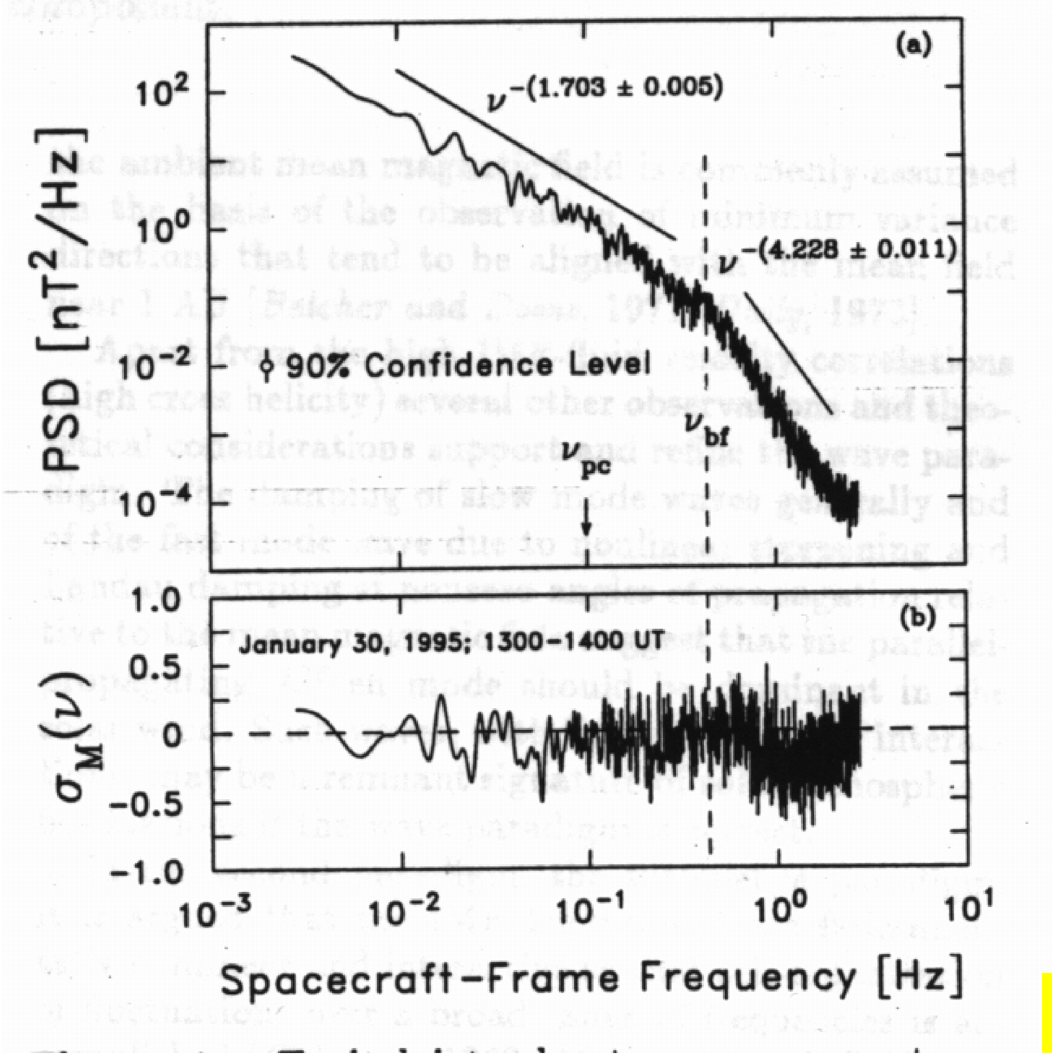
$\dot{Q}_p(\mathbf{k})$: Rate of wave generation.

$T^W_{\rm esc}$: Wave leakage timescale.

$\gamma(k) = \gamma_e + \gamma_p$: The damping coefficients.

D_{ij}: Wave diffusion tensor.

Magnetic fluctuations in Solar wind



Magnetic fluctuations in Solar wind

Leamon et al (1998)

4. Interactions with Particles: *Heating and Acceleration*

Resonant Wave-Particle Interactions

Interaction Rates

Dispersion Relations

Particle Kinetic Equation

Wave-Particle Interaction Rates

- Dominated by Resonant Interactions

$$D_{ij} = \pi e^2 \sum_{n=-\infty}^{+\infty} \int d^3k \langle d_{ij} \rangle \delta\left(\mathbf{k} \cdot \mathbf{v} - \omega + \frac{n\eta_0}{\gamma} \Omega_0 \right),$$

- Lower energy particles interacting with higher wavevectors or frequencies

Dispersion Relation for the Waves
(Propagating Along Field Lines)

$$(ck)^2 = \omega^2 \left[1 - \sum_i \frac{\omega_{pi}^2}{\omega(\omega - q_i/|q_i|\Omega_i)} \right].$$

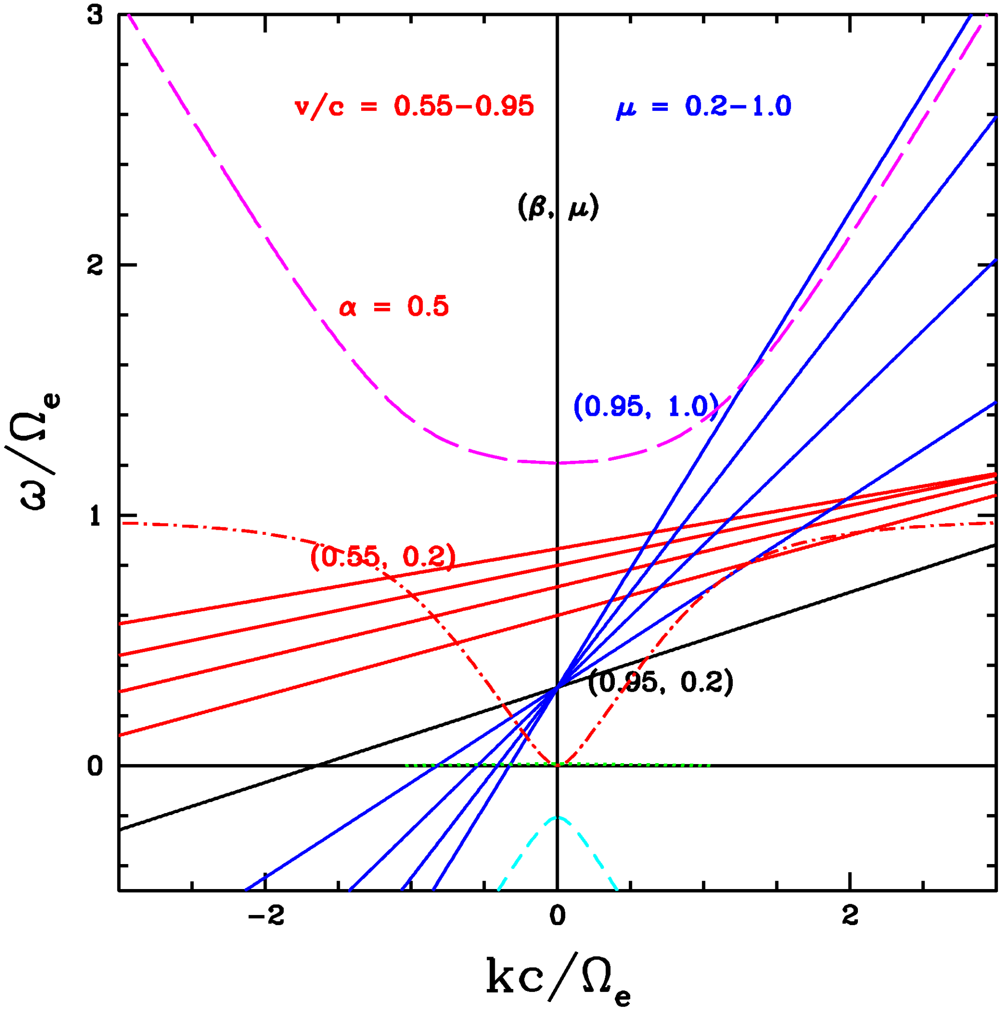
Plasma Parameter:

$$\alpha = \frac{\omega_{pe}}{\Omega_e} = 1.0 \left(\frac{n}{10^9 \mathrm{cm}^{-3}} \right)^{1/2} \left(\frac{B_0}{100\mathrm{G}} \right)^{-1}$$

Abundances: Electrons, protons and alpha particles

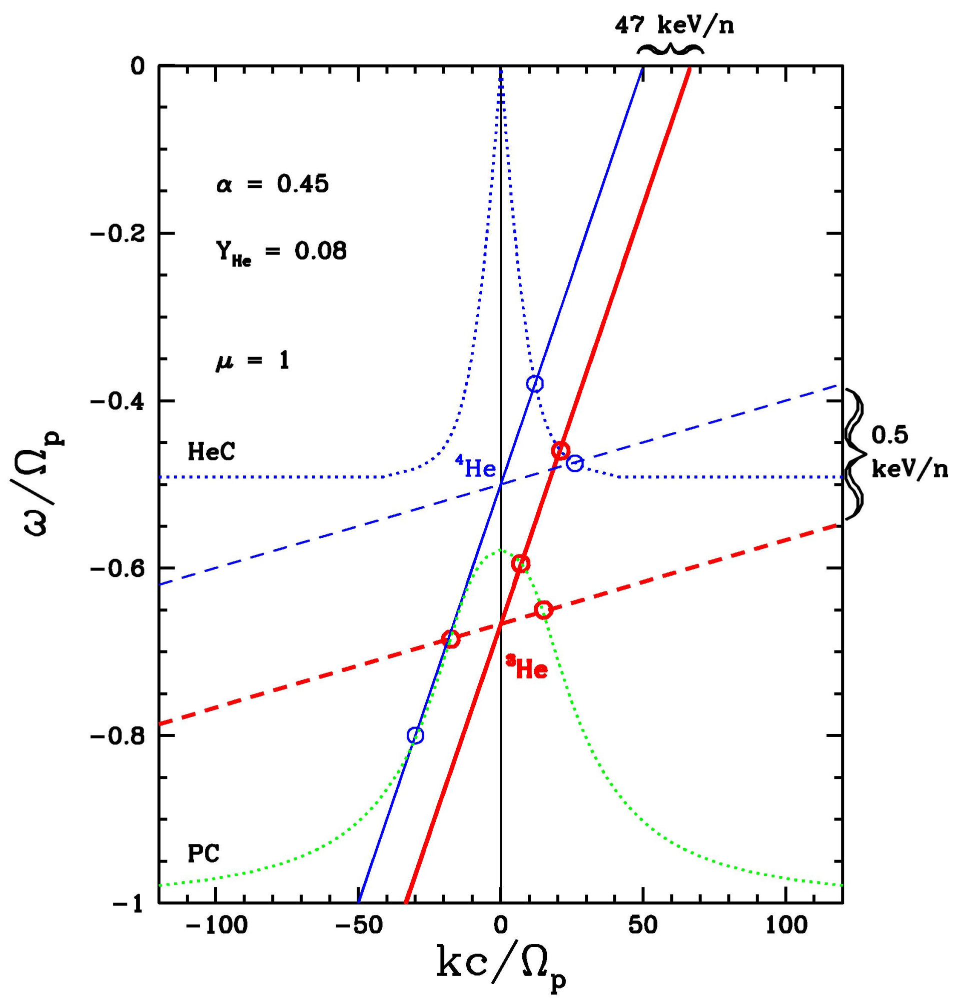
Resonant Interaction *electrons*

$$\omega = \mu v k + \frac{\Omega_i}{\gamma}.$$

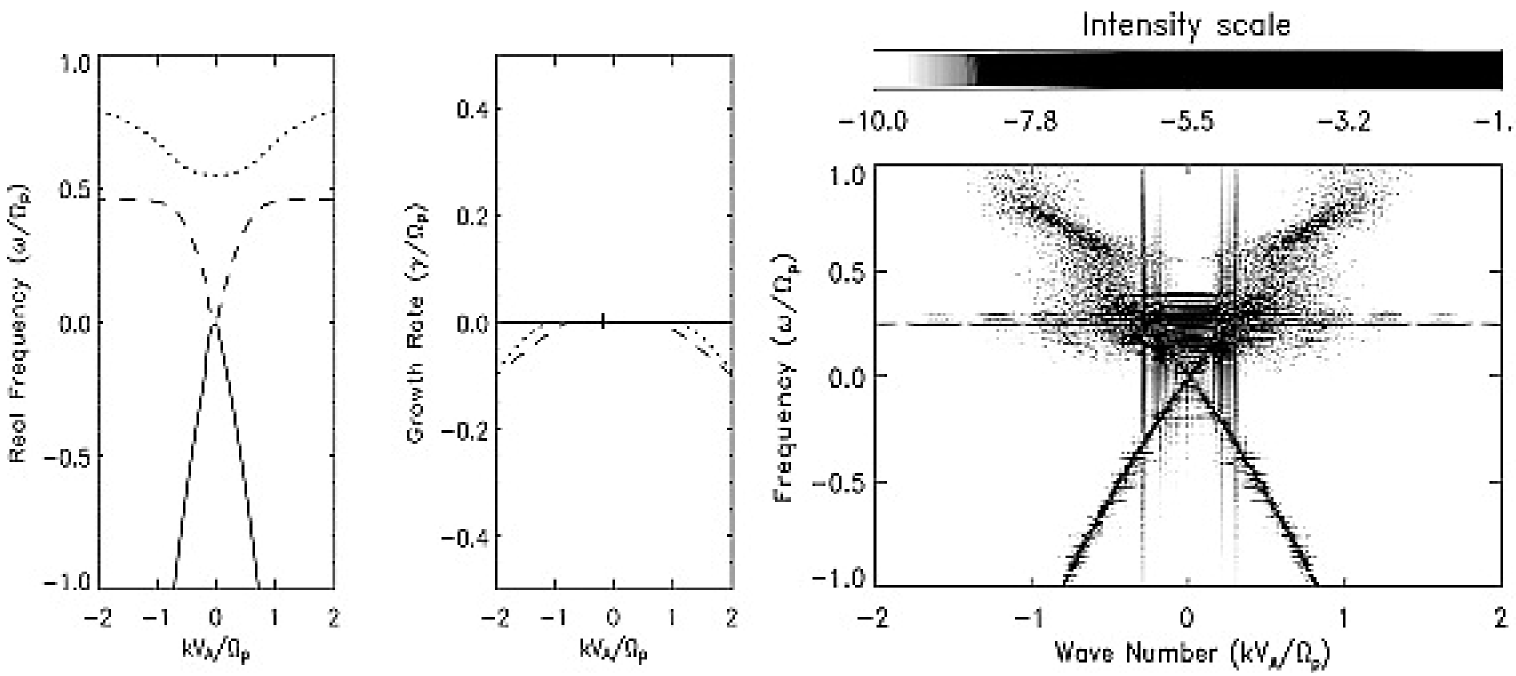


Resonant Wave-Particle Interactions
4He and 3He

$$\omega = \mu v k + \frac{\Omega_i}{\gamma}.$$

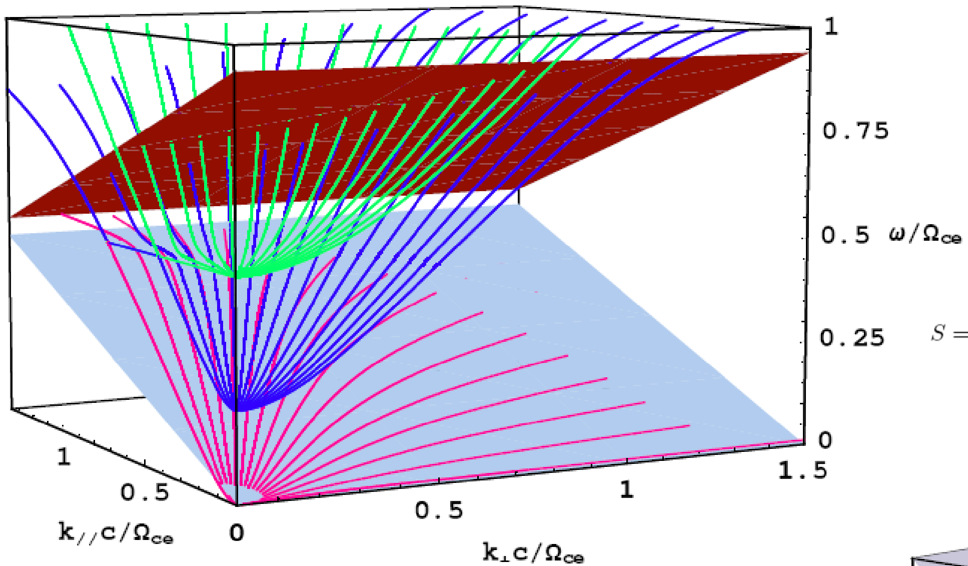


Simulations of The Wave Modes



- From Opher et al.

General Dispersion Relation



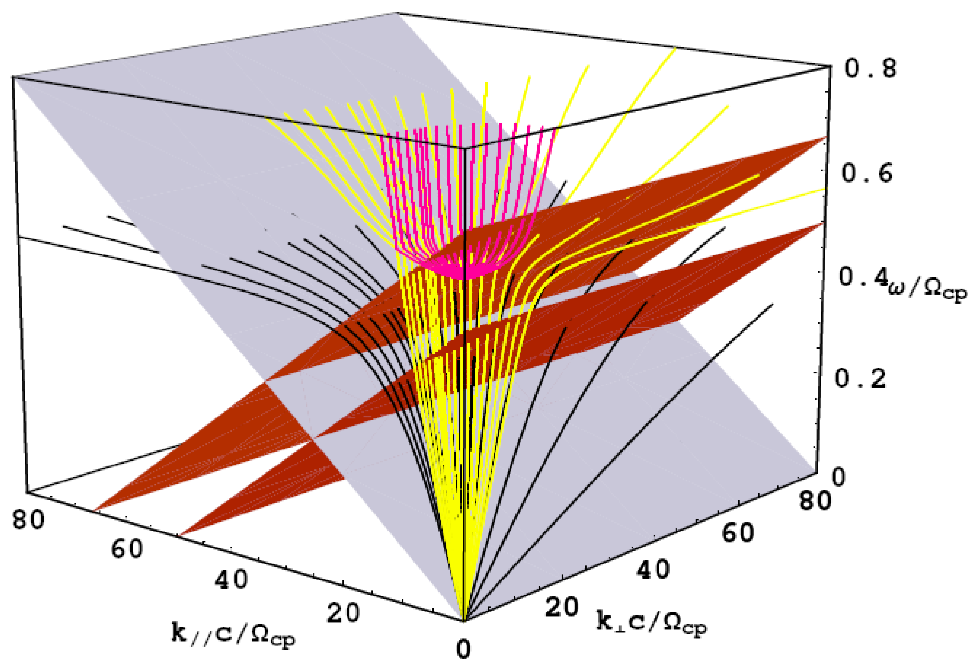
$$\tan^2\theta = \frac{-P(n^2 - R)(n^2 - L)}{(Sn^2 - RL)(n^2 - P)}$$

$$S = \frac{1}{2}(R + L), \qquad D = \frac{1}{2}(R - L), \qquad P = 1 - \sum_i \frac{\omega_{pi}^2}{\omega^2}, \qquad R = 1 - \sum_i \frac{\omega_{pi}^2}{\omega^2}\left(\frac{\omega}{\omega + \epsilon_i \Omega_i}\right),$$

$$L = 1 - \sum_i \frac{\omega_{pi}^2}{\omega^2}\left(\frac{\omega}{\omega - \epsilon_i \Omega_i}\right), \qquad \omega_{pi}^2 = \frac{4\pi n_i q_i^2}{m_i}, \qquad \Omega_i = \frac{|q_i| B}{m_i c}, \qquad \epsilon_i = \frac{q_i}{|q_i|}$$

Resonance Condition

$$\omega = k_\parallel v \mu + n\Omega_i/\gamma$$



5. PARTICLE SPECTRA

ISOTROPIC AND HOMOGENEOUS

$$\frac{\partial N}{\partial t} = \frac{\partial^2}{\partial E^2}(D_{EE}N) + \frac{\partial}{\partial E}[(\dot{E}_{\mathrm{L}} - A)N] - \frac{N}{T_{\mathrm{esc}}} + Q$$

$$A(E) = \frac{\mathrm{d}D_{EE}}{\mathrm{d}E} + D_{EE}\frac{2\gamma^2 - 1}{(\gamma^2 - 1)\gamma m c^2} \quad + A_{shock}$$

$$T_{\mathrm{esc}} = \frac{L}{\sqrt{2}v}\left(1 + \frac{\sqrt{2}L}{v\tau_{\mathrm{sc}}}\right) \qquad \tau_{\mathrm{sc}} = \frac{1}{2}\int_{-1}^{1}\mathrm{d}\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}}$$

A. KINETIC EQUATION

Liouville or Boltzmann equation in limit of many "small" scatterings leads to

The General Fokker-Planck equation:

$$\frac{\partial f}{\partial t} + v\mu\frac{\partial f}{\partial s} = \frac{1}{p^2}\frac{\partial}{\partial p}p^2\left[D_{pp}\frac{\partial f}{\partial p} + D_{p\mu}\frac{\partial f}{\partial \mu}\right] + \frac{\partial}{\partial \mu}\left[D_{\mu\mu}\frac{\partial f}{\partial \mu} + D_{\mu p}\frac{\partial f}{\partial p}\right] - \frac{1}{p^2}\frac{\partial}{\partial p}(p^2\dot{p}_L f) + S\,.$$

$f(p, \mu, s, t)$; gyrophase averaged particle distribution

s is the distance along the background B field

S is a source term

1. Isotropic, High Energy Limit:

$D_{\mu\mu} >> v/L$ and D_{pp}/p^2

$$F(p, s, t) \equiv \frac{1}{2}\int_{-1}^{1} \mathrm{d}\mu f(p, \mu, s, t),$$

$$\frac{\partial F}{\partial t} - \frac{\partial}{\partial z}\kappa_1\frac{\partial F}{\partial z} = (pv)\frac{\partial \kappa_2}{\partial z}\frac{\partial F}{\partial p} - \frac{1}{p^2}\frac{\partial}{\partial p}(p^3 v\kappa_2)\frac{\partial F}{\partial z} + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^4\kappa_3\frac{\partial F}{\partial p} - p^2\dot{p}_L F\right) + Q(p, s, t)\,,$$

$$\kappa_1 = \frac{v^2}{8}\int_{-1}^{1}\mathrm{d}\mu\frac{(1-\mu^2)^2}{D_{\mu\mu}}, \quad \kappa_2 = \frac{1}{4}\int_{-1}^{1}\mathrm{d}\mu(1-\mu^2)\frac{D_{\mu p}}{pD_{\mu\mu}}$$
$$\kappa_3 = \frac{1}{2}\int_{-1}^{1}\mathrm{d}\mu(D_{pp} - D_{\mu p}^2/D_{\mu\mu})p^2, \quad Q(p, s, t) \equiv \frac{1}{2}\int_{-1}^{1}\mathrm{d}\mu S(p, \mu, s, t).$$

The acceleration and scattering times are

$$\tau_{ac} = 1/\kappa_3 \quad \tau_{sc} = 8\kappa_1/v^2.$$

COUPLED EQUATIONS

1. Kinetic Equations

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left[D_{EE}\frac{\partial N}{\partial E} - (A - \dot{E}_L)N\right] - \frac{N}{T^p_{\rm esc}} + \dot{Q}^p$$

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial k_i}\left[D_{ij}\frac{\partial}{\partial k_j}W\right] - \Gamma(\mathbf{k})W - \frac{W}{T^W_{\rm esc}(\mathbf{k})} + \dot{Q}^W$$

2. Energy Balance

$$\dot{W}_{nonth} \equiv \int \Gamma_{nonth}(\mathbf{k})W(\mathbf{k})d^3k = \dot{\mathcal{E}} \equiv \int A(E)N(E)dE$$

3. Rate Coefficients

$$A(E) = \frac{d[vp^2D(p)]}{4p^2dp} = \int_{k_{min}}^{\infty} d^3kW(\mathbf{k})\Sigma(\mathbf{k},E)$$

$$\Gamma_{nonth}(\mathbf{k}) = \int_{E_0}^{\infty} dEN(E)\Sigma(\mathbf{k},E)$$

5. Accelerated Particle Spectra
Model Parameters

In principle:	Density	n
	Temperature	T
	Magnetic Field	B
	Scale (geometry)	L
	Level of Turbulence	$(\delta B / B)^2$

5. Accelerated Particle Spectra
Kinetic Equation Coefficients

Acceleration rate or time τ_{ac}

Loss rate or time τ_{loss}

Escape rate or time T_{esc}

Characteristic Times:

$$\tau_p^{-1} \propto \Omega_e (\delta B / B)^2 \text{ and } T_{cross} \approx L / v$$

Some Attractive Features

1. Acceleration of Background Particles
2. Spectral Breaks
3. Heating and Acceleration
4. Proton (or ion) vs Electron Acceleration
5. Effects of **shocks** can be included

A SIMPLE EXAMPLE

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial \gamma}\left[\frac{\partial \gamma^2 N}{\partial \gamma} - \left(4\gamma - \frac{4\gamma^2 \tau_{\rm ac}}{\tau_0}\right) N\right] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

$$\tau_{\rm ac} = \frac{C_1}{f_{\rm turb}} \frac{cR}{v_{\rm A}^2}$$

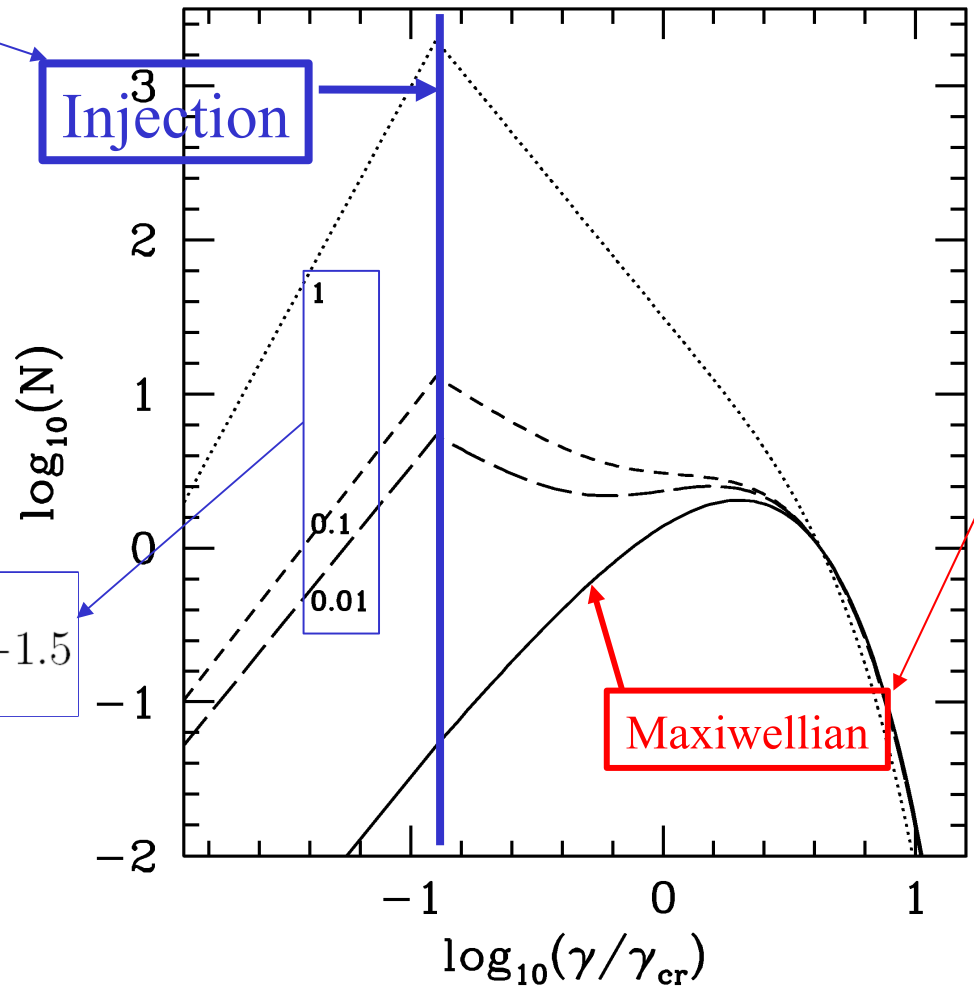
$$\tau_{\rm syn}(\gamma) = 9m_e^3c^5/4e^4B^2\gamma = \tau_0/\gamma$$

$$\gamma_{cr} = \frac{\tau_0}{4\tau_{\rm ac}} = \frac{9m_e^3c^4v_{\rm A}^2 f_{\rm turb}}{16e^4RB^2C_1} = 30\left(\frac{R}{r_S}\right)^{-1}\left(\frac{n}{10^7\ {\rm cm}^{-1}}\right)^{-1}\left(\frac{f_{\rm turb}}{C_1}\right)$$

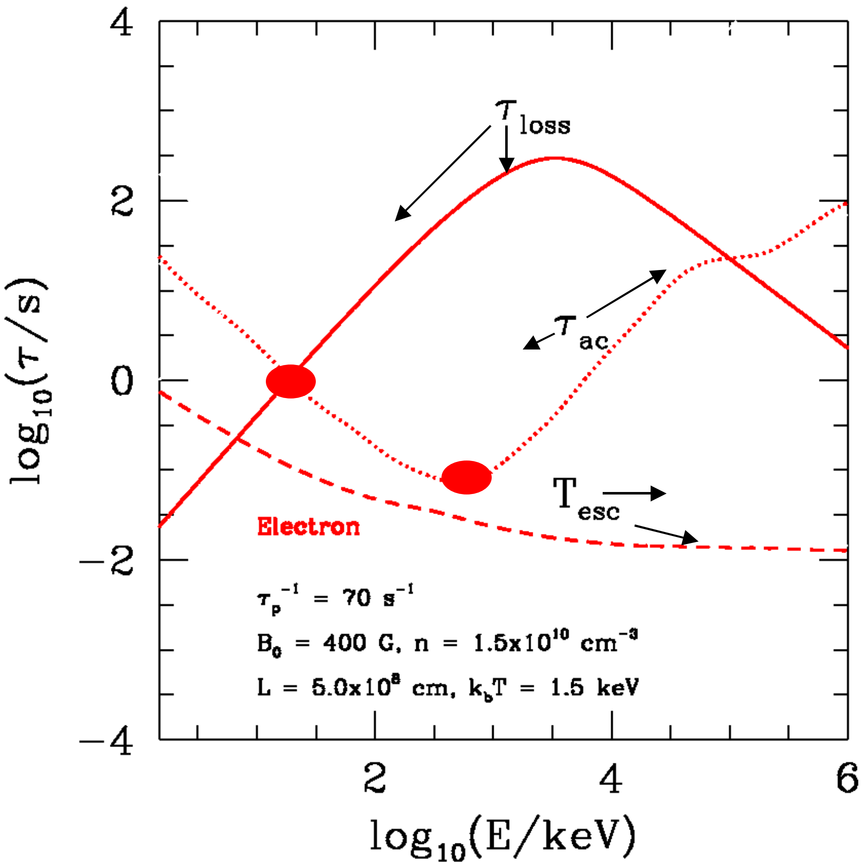
Electron Spectra and γ_{min} =?

Continuous injection, heating, & cooling.

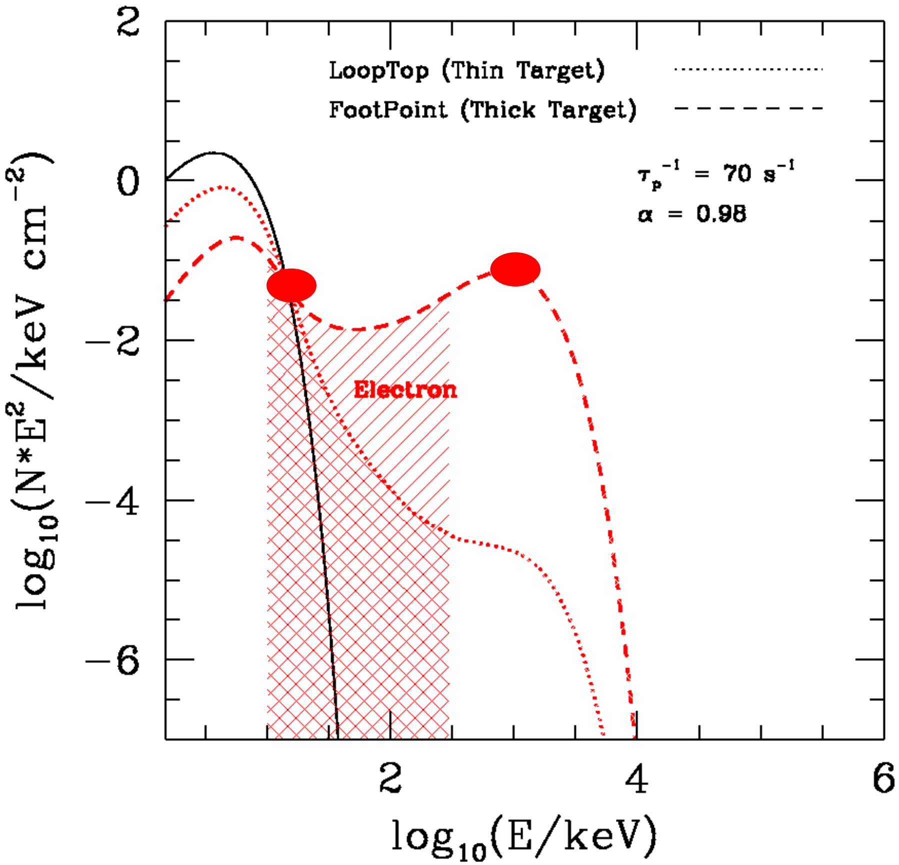
$$\delta = \left(\frac{9}{4} + \frac{2\tau_{ac}}{T_{esc}}\right)^{1/2} - 1.5$$



Without injection: Continuous heating and cooling

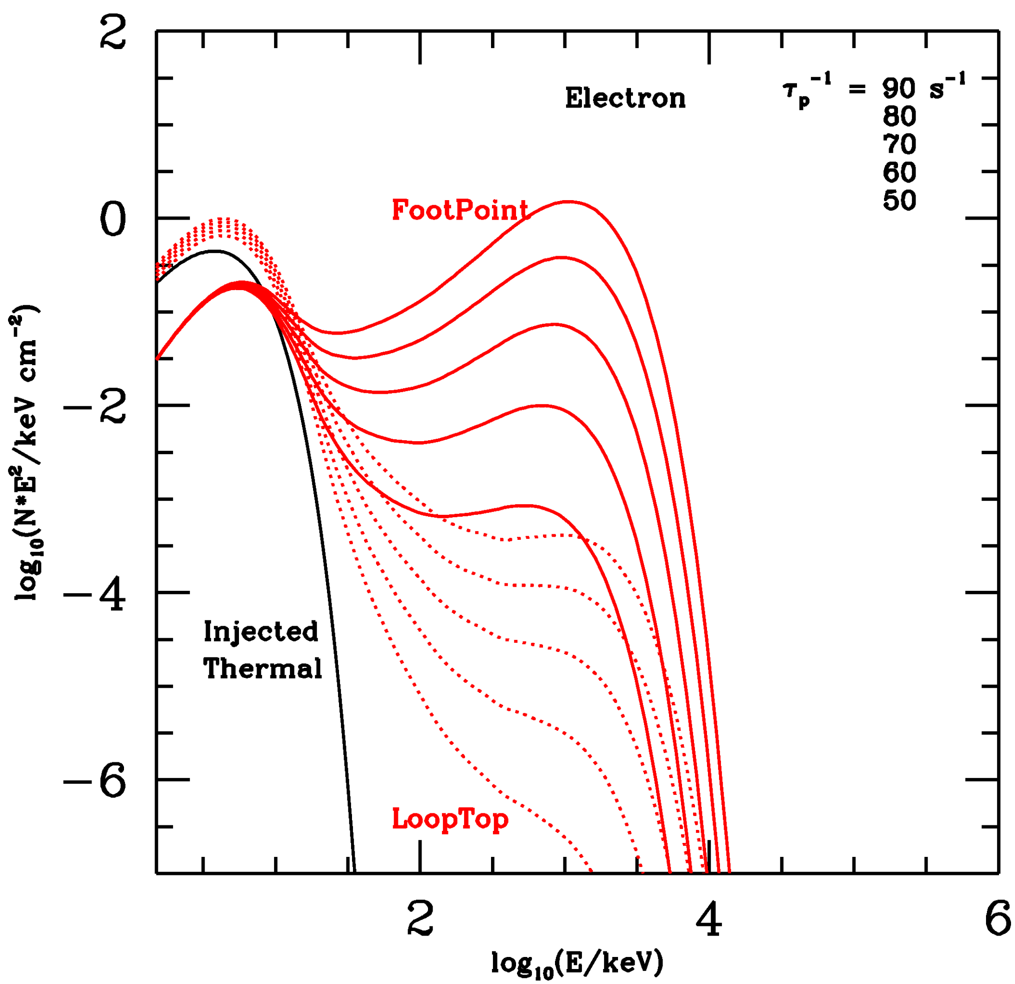


Time Scales



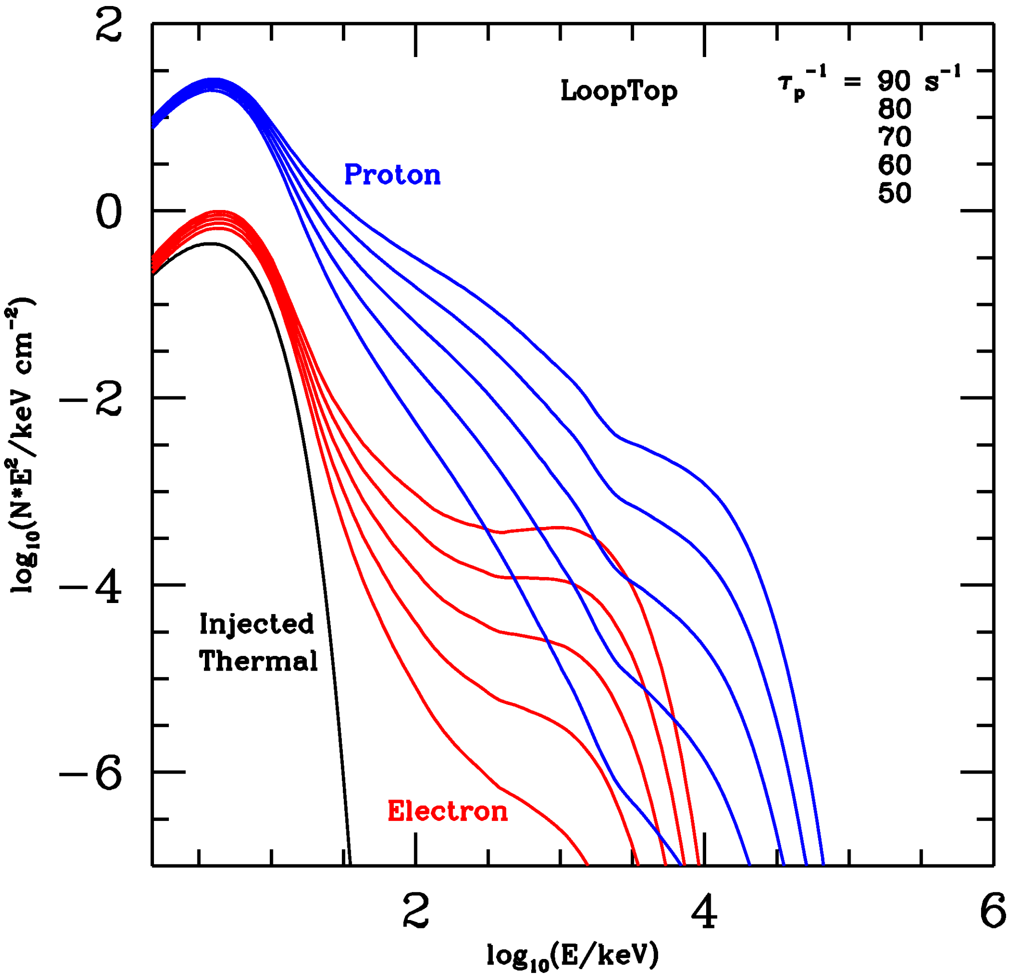
Steady State Particle Distributions

SPECTRAL HARDNESS

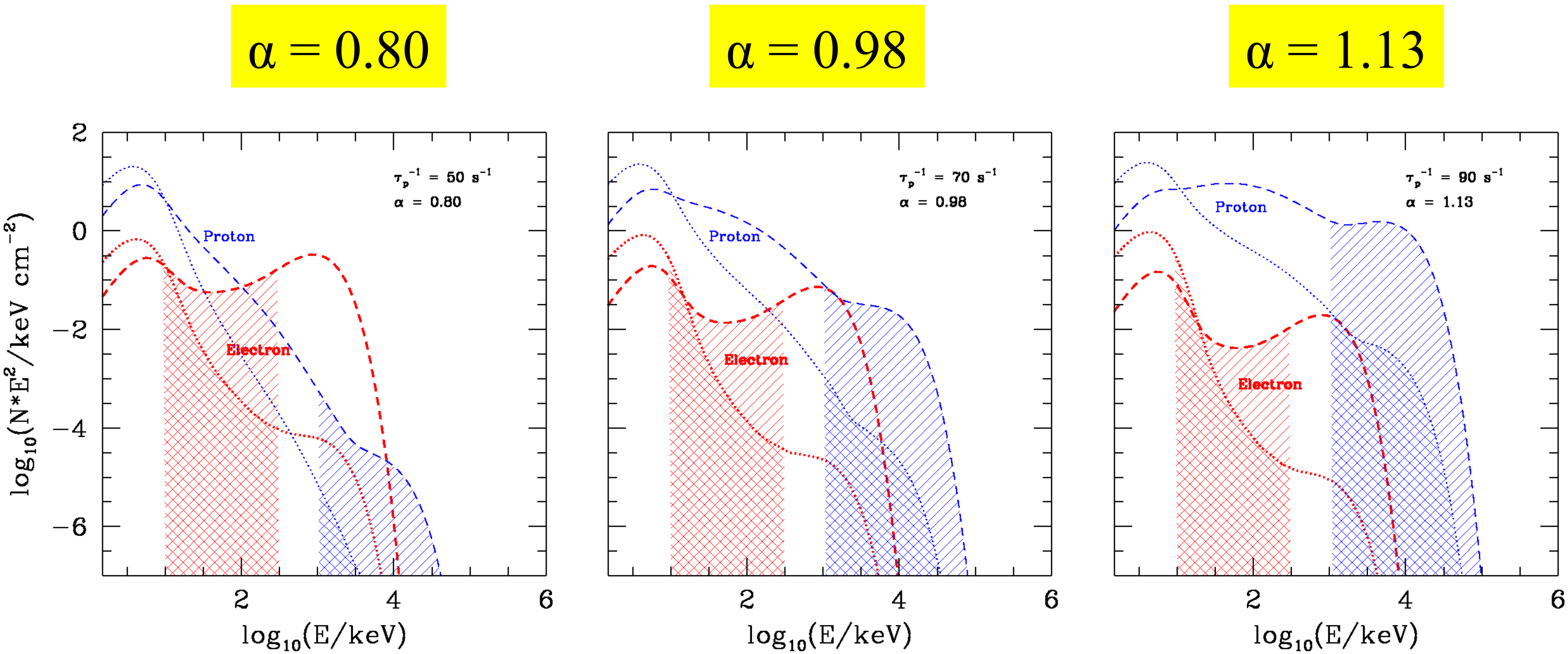


HEATING VS ACCELERATION

Electron vs Proton Acceleration

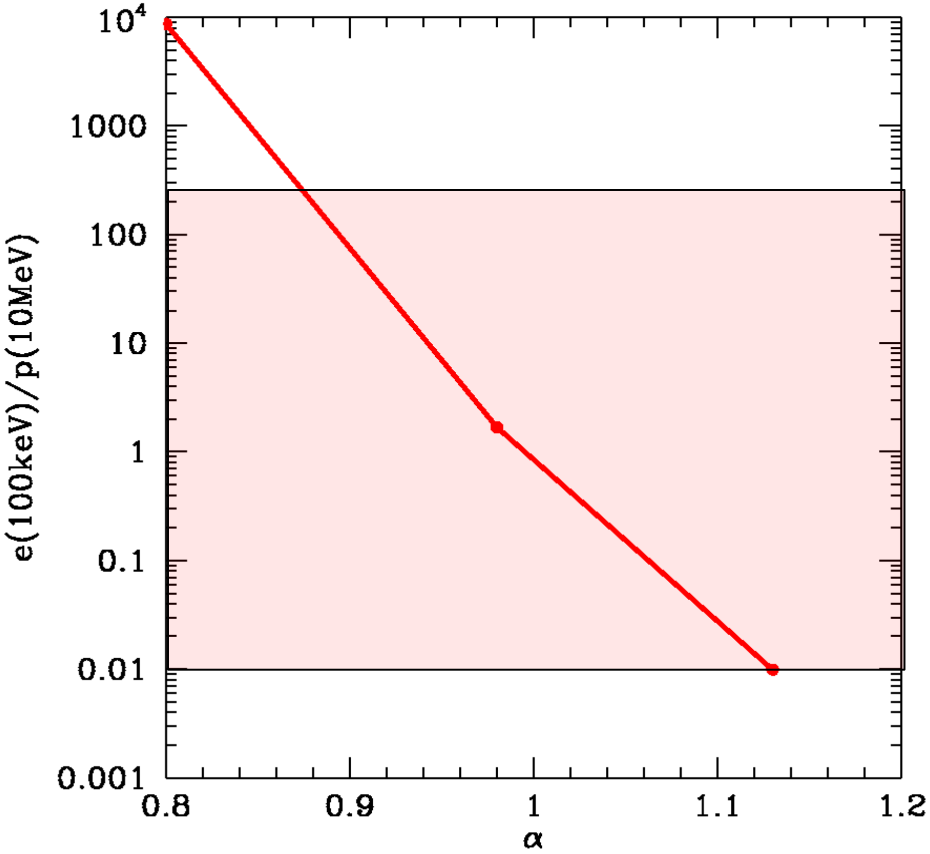


Protons vs. Electrons



Dependence on the Plasma Parameter

Protons vs. Electrons



SUMMARY

- Turbulence and plasma waves play major roles in non-thermal sources in energizing the plasma and accelerating particles.
- These are the dominant acceleration process at low energies and scattering agent at all energies.
- It can describe many features of radiation and particle spectra from a variety of sources.

III. Some Applications

Solar Flares

Sgr A* (slow accreting AGNs)

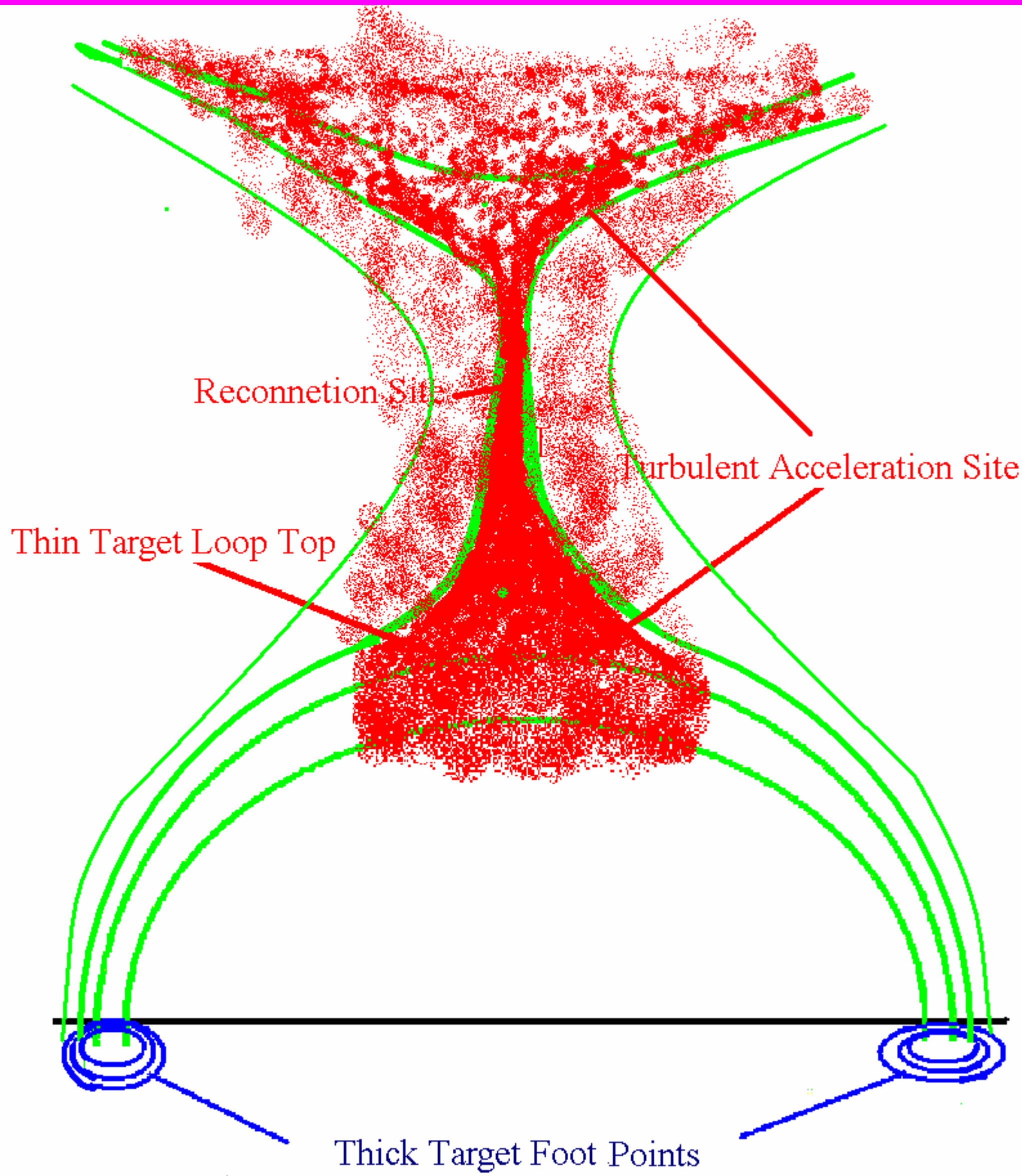
SOLAR FLARES

1. Electron Acceleration and Emission

2. Proton and Ion Acceleration and Emission

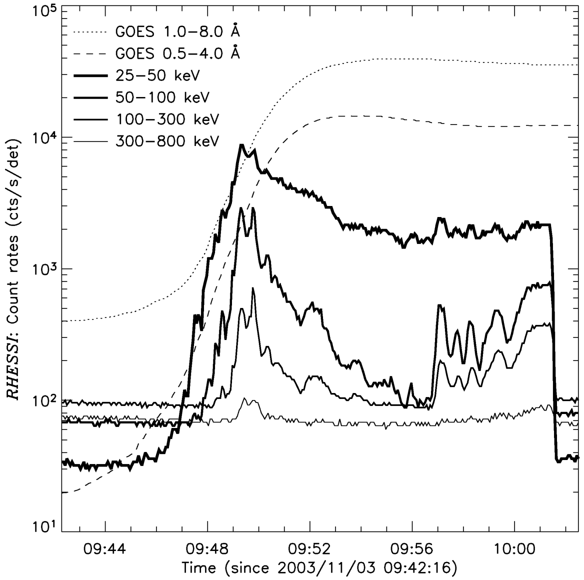
3. Solar Cosmic Rays (SEPs)

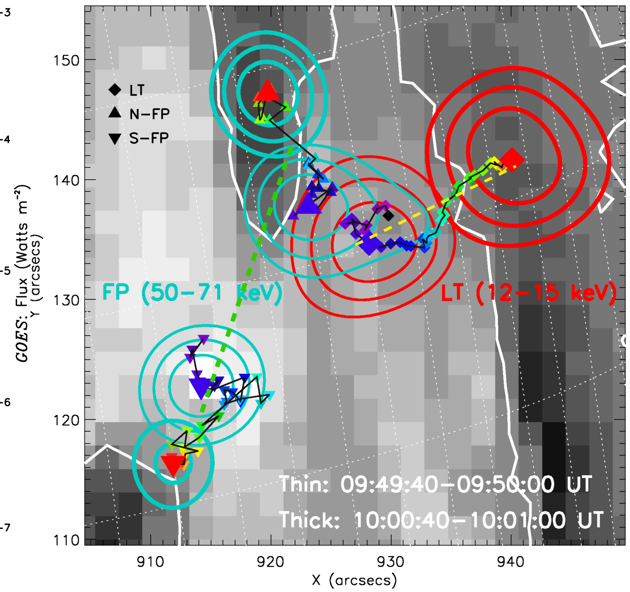
Model Description

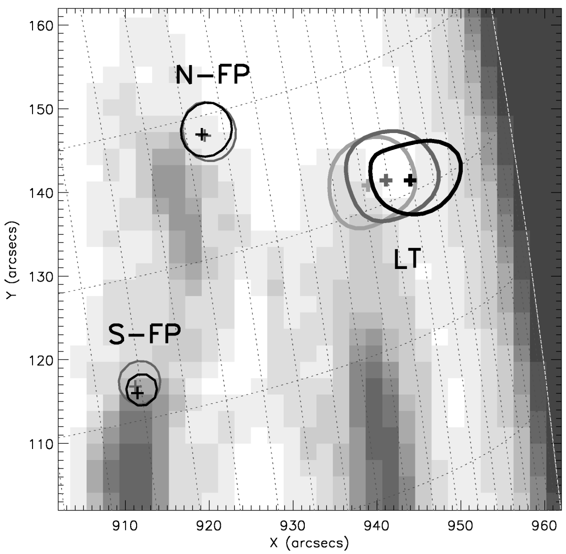


A Simple Solar Flare

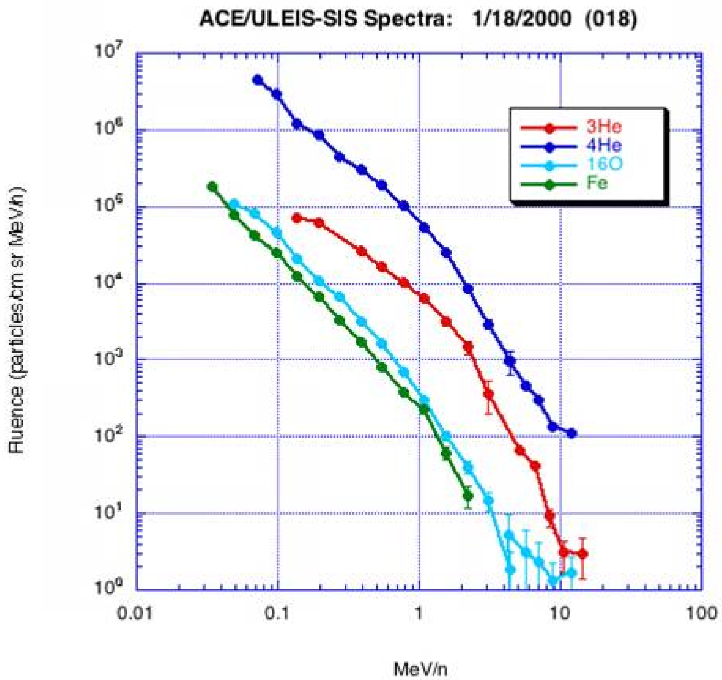
11032003, N09W77, X3.9

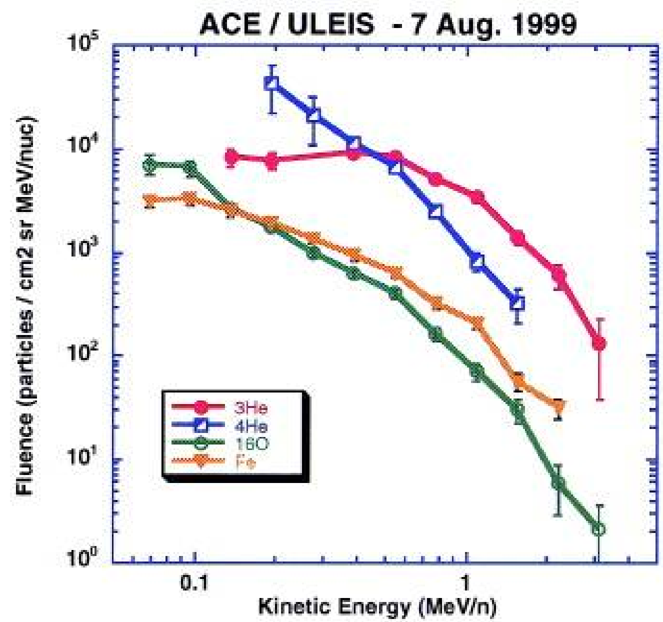


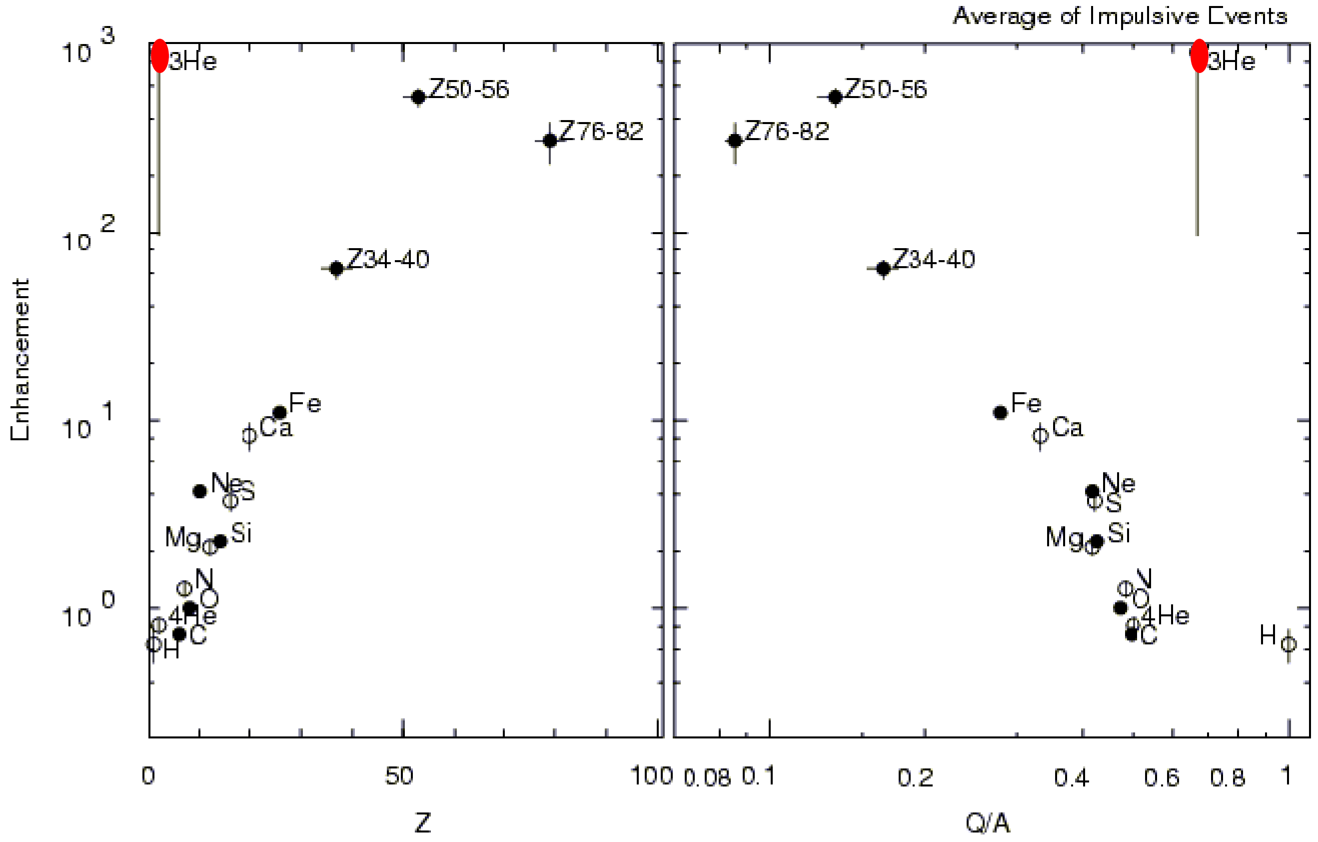




Event #11, Mason et al., ApJ, 574, 1039, 2002

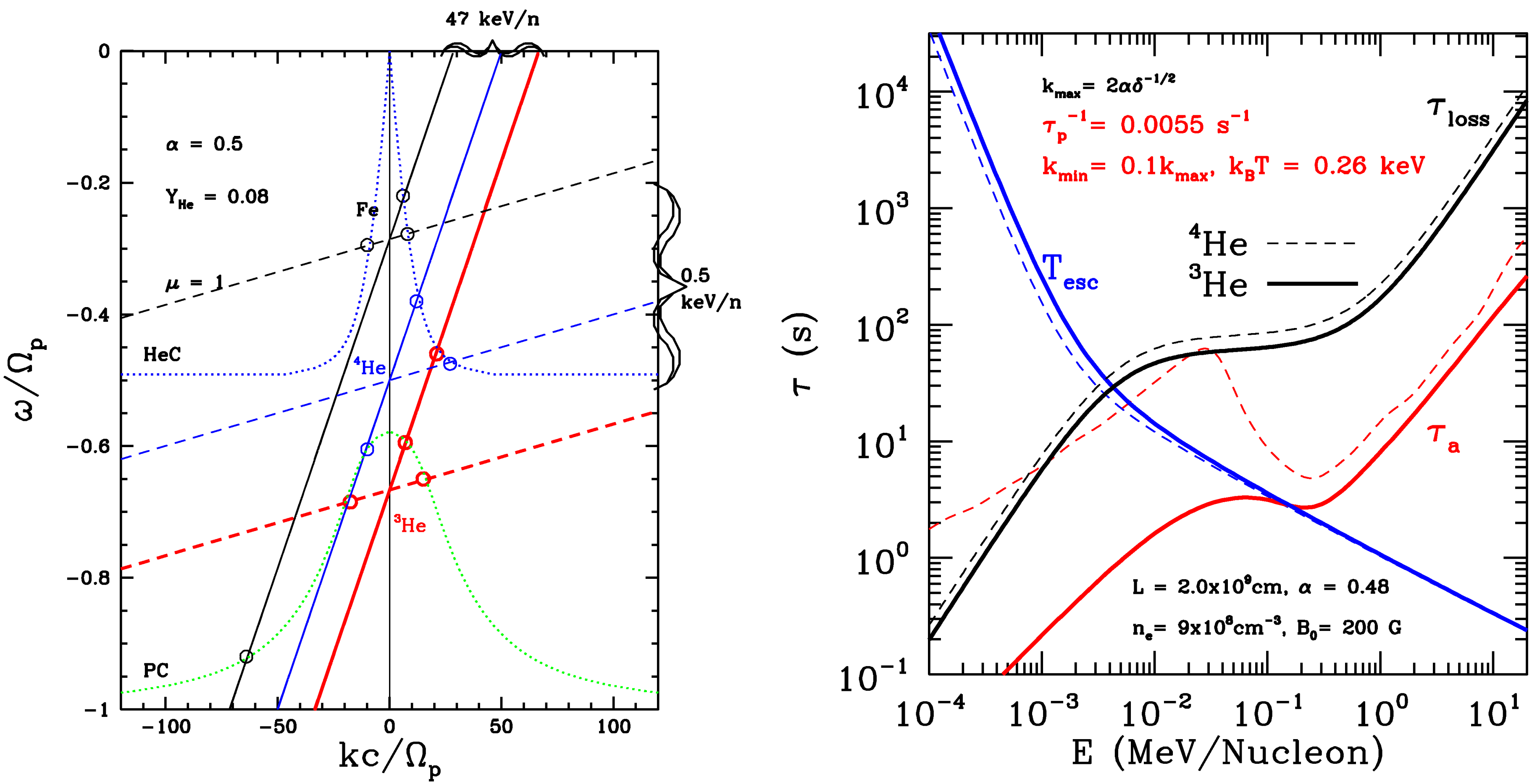




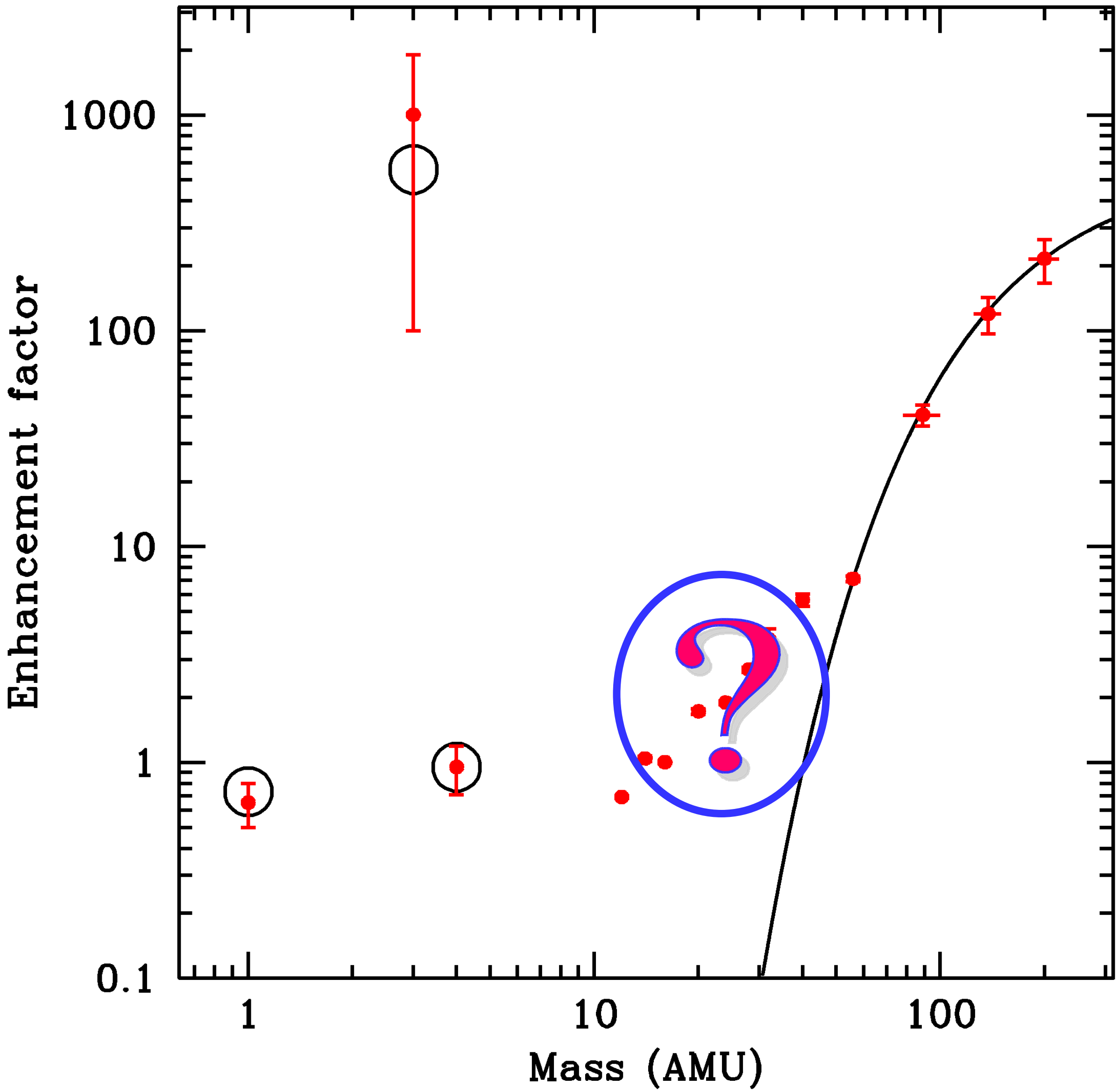


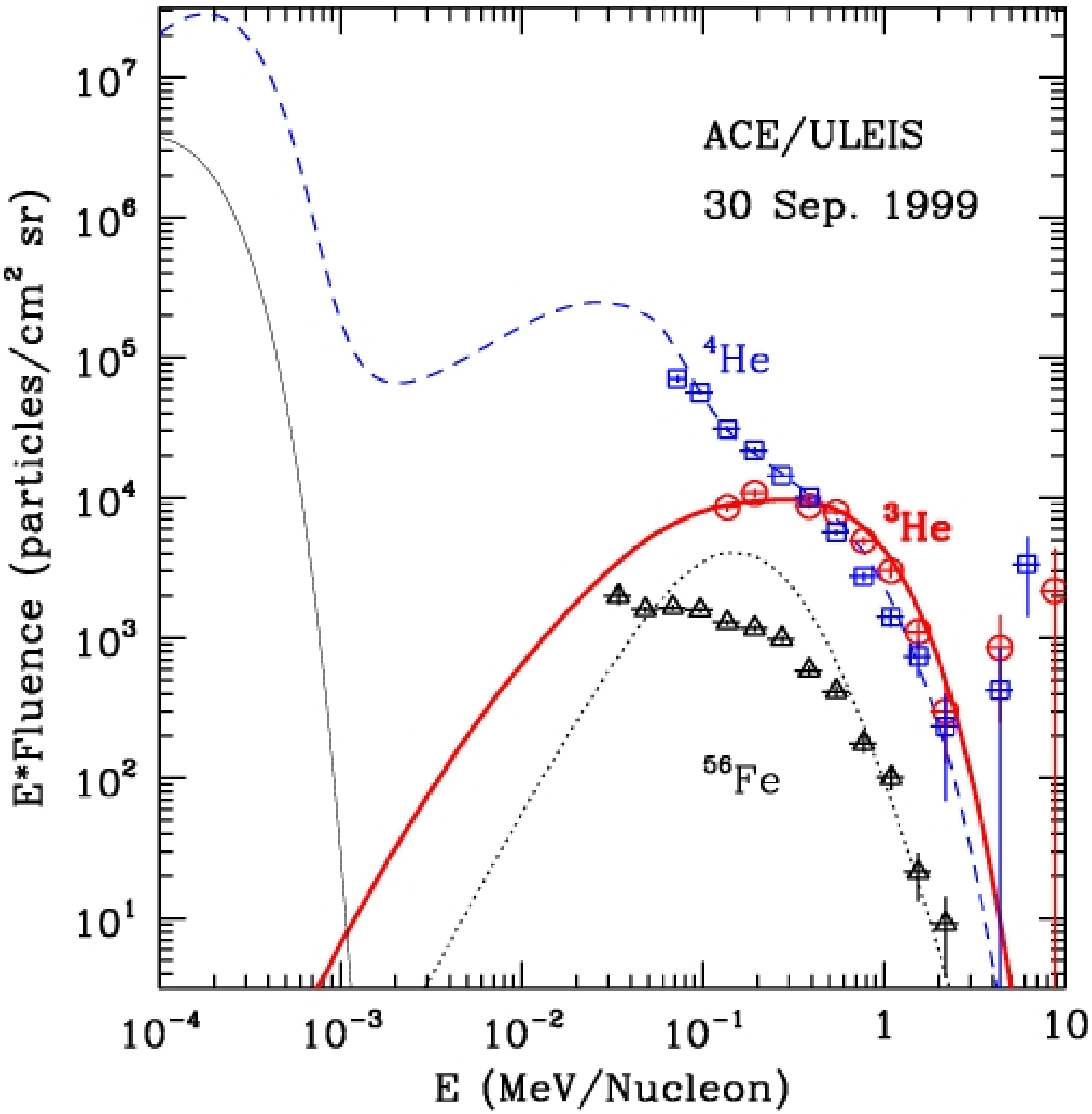
Reames and Ng 2004

Acceleration of ^{3}He and ^{4}He by Parallel Propagating Waves



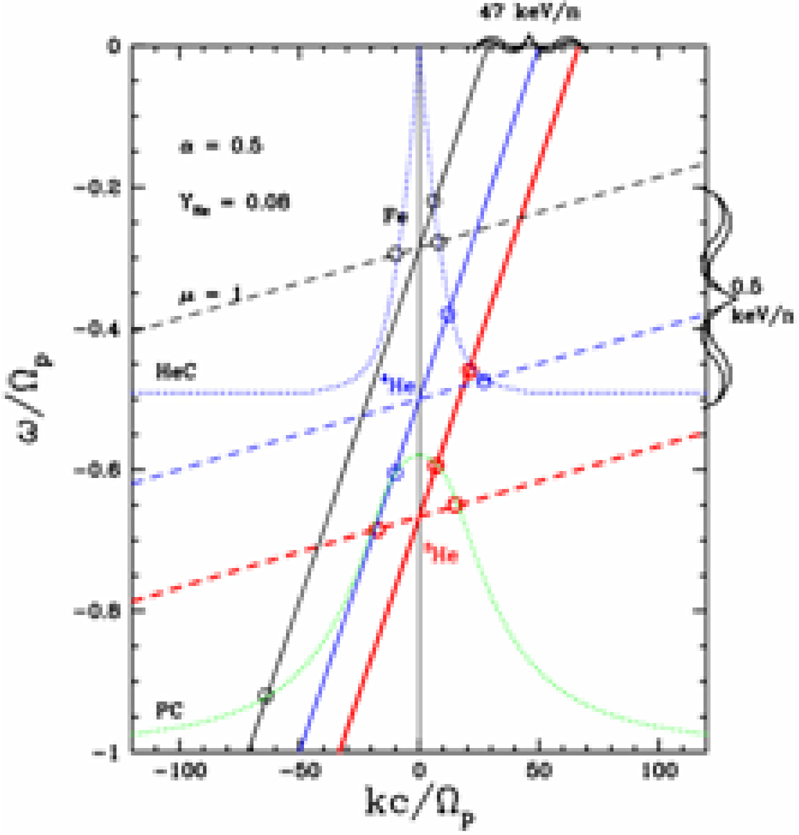
3He and Heavy Ion Enrichment

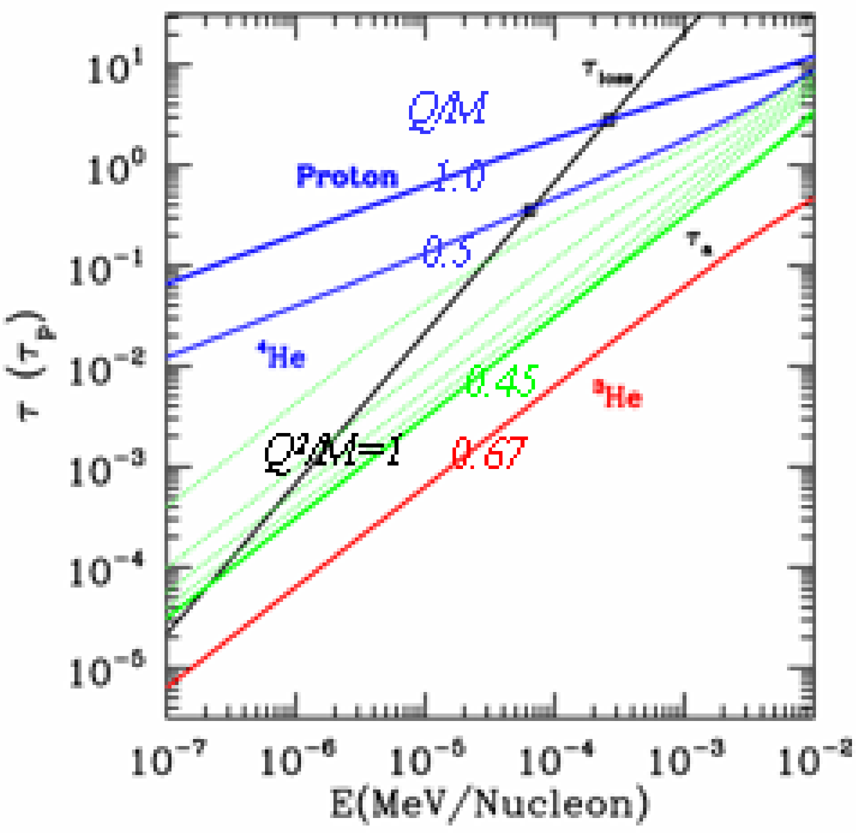




ACE/ULEIS
30 Sep. 1999
^{4}He
^{3}He
^{56}Fe
E*Fluence (particles/cm^2 sr)
E (MeV/Nucleon)
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10

3. Ion Acceleration by Parallel Propagating Waves





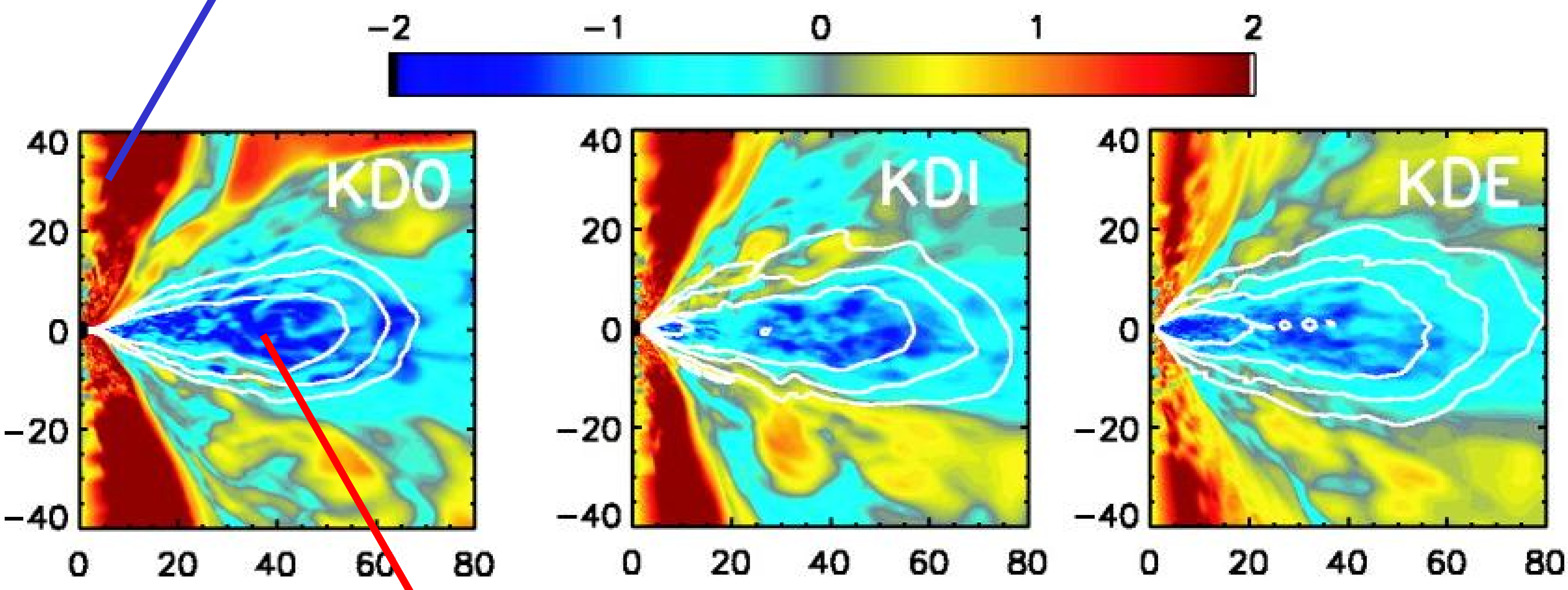
Sgr A*

Proton and Electron Acceleration
in the Galactic Center HESS Source

Electron Acceleration
During the NIR and X-ray Flares

Structure of the Accretion Flow

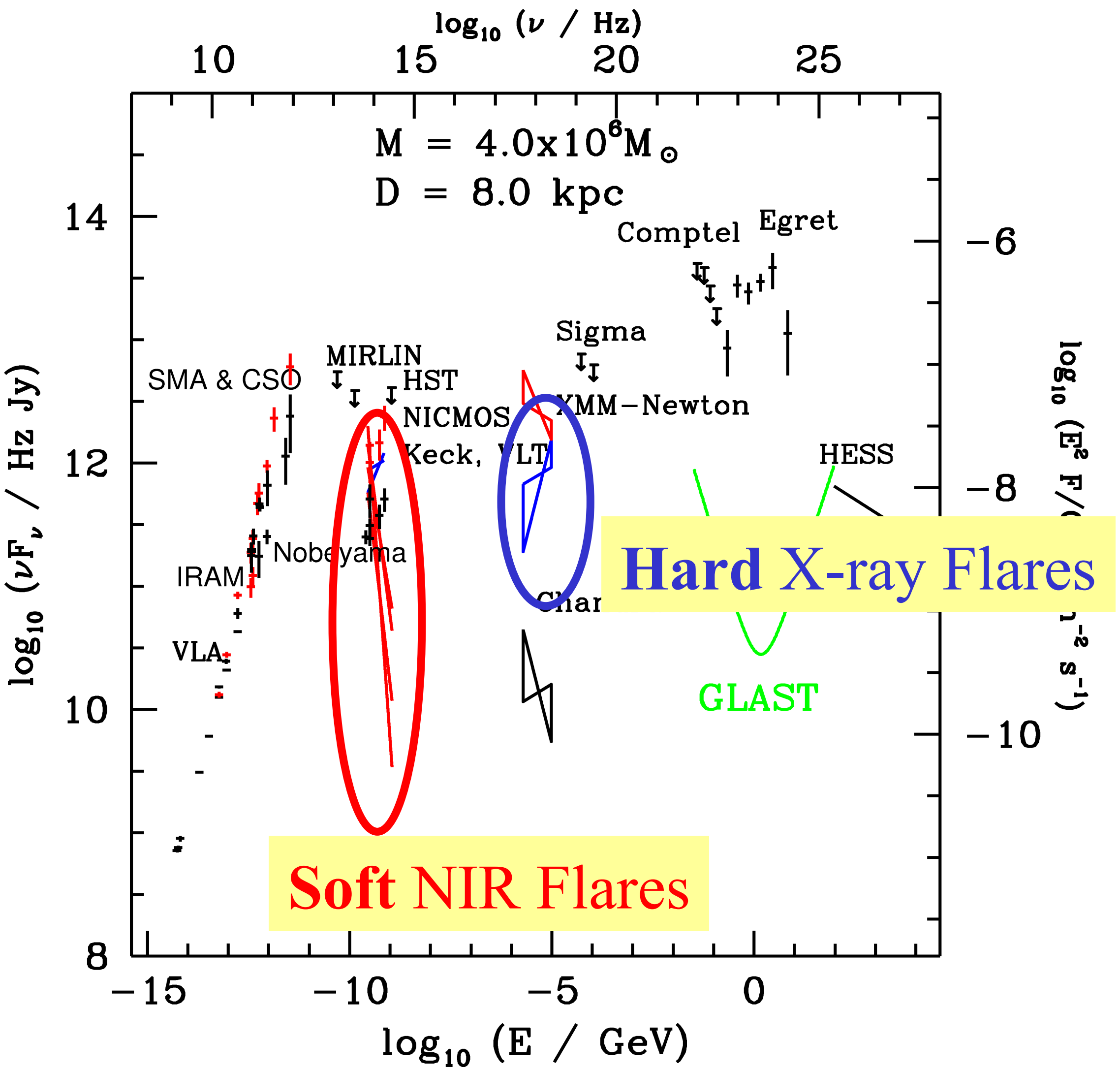
cm and mm via Synchrotron and proton acceleration



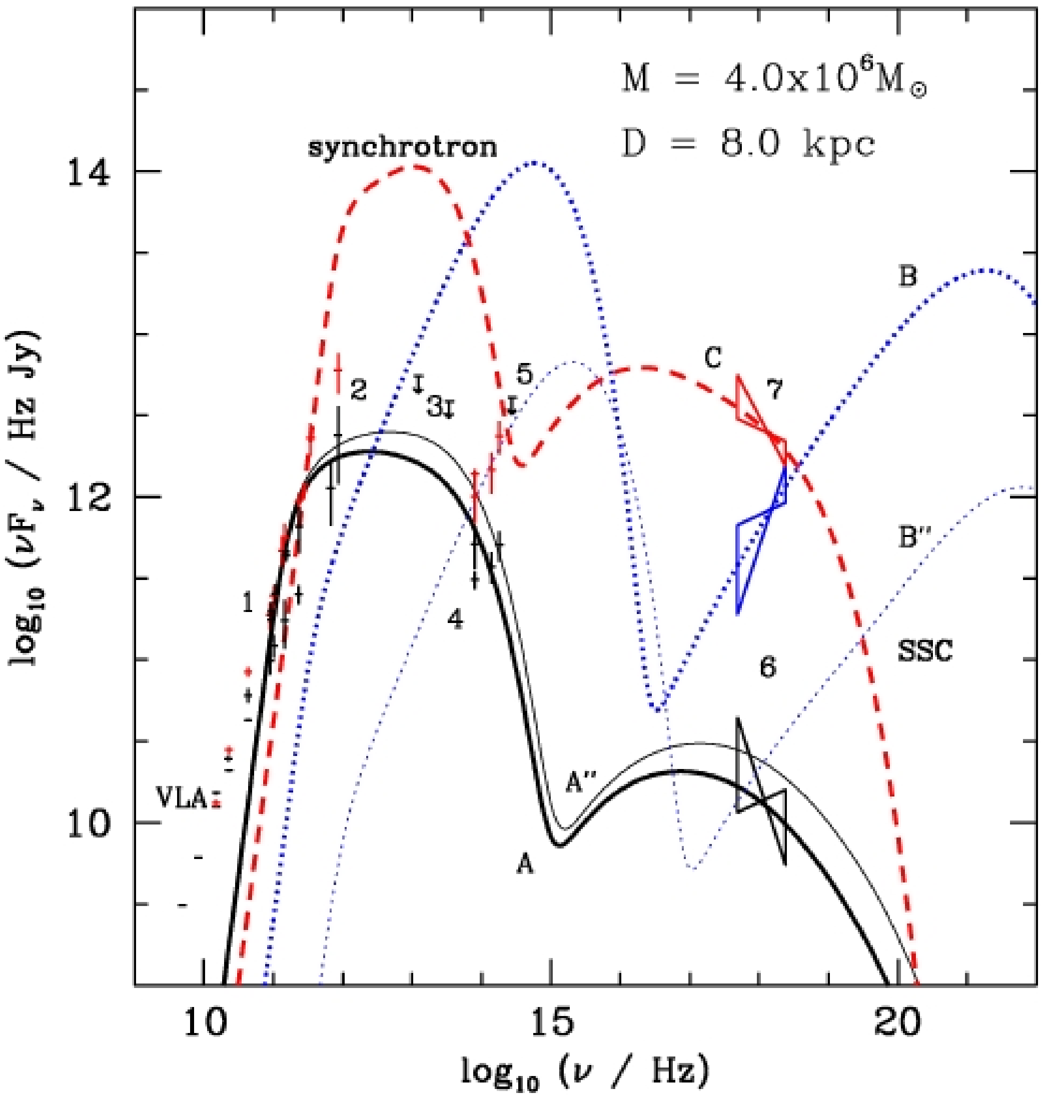
Sub-mm, NIR, and X-ray via Synchrotron and SSC

De Villiers et al. 2003 ApJ

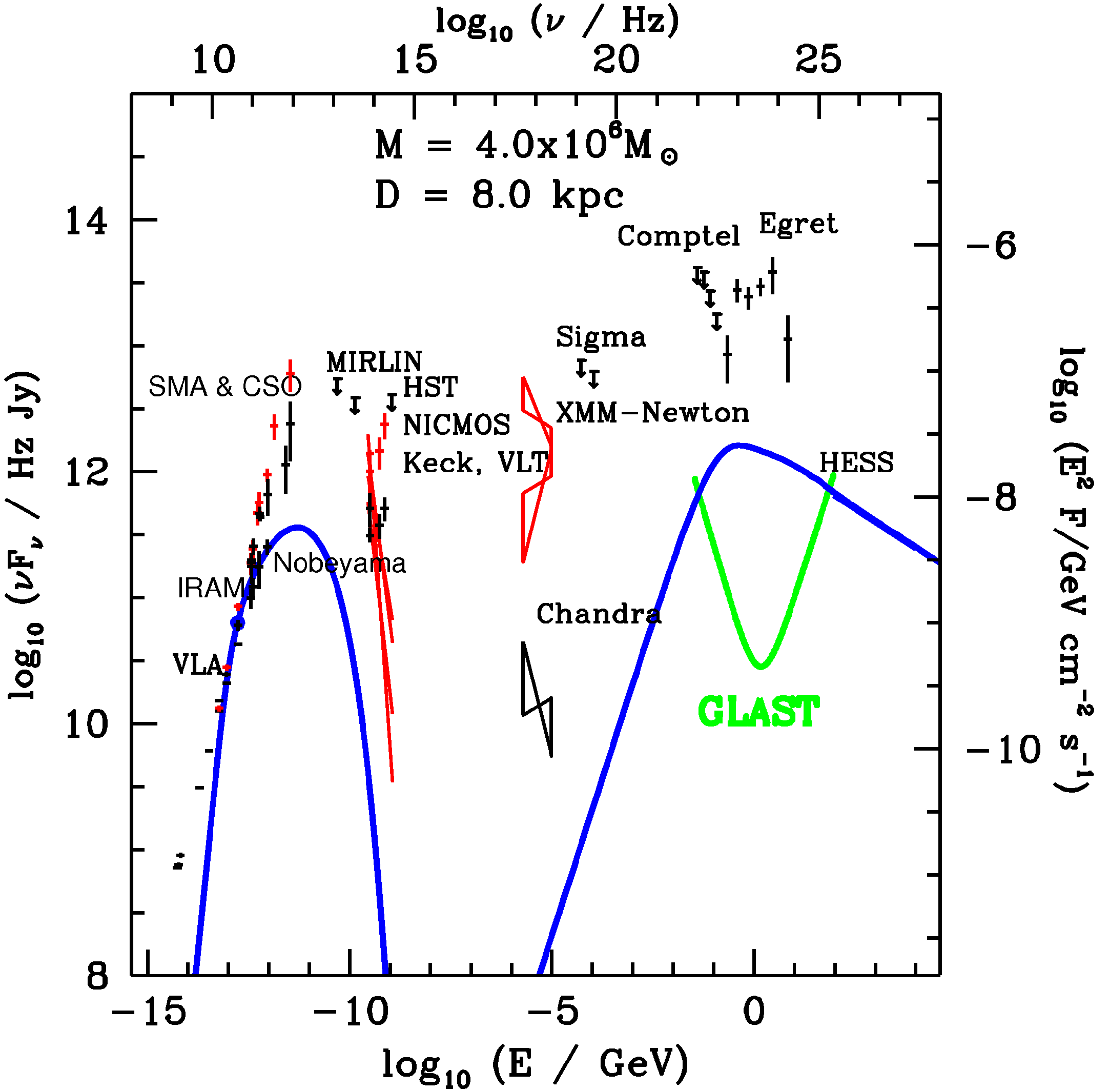
Broadband Spectrum



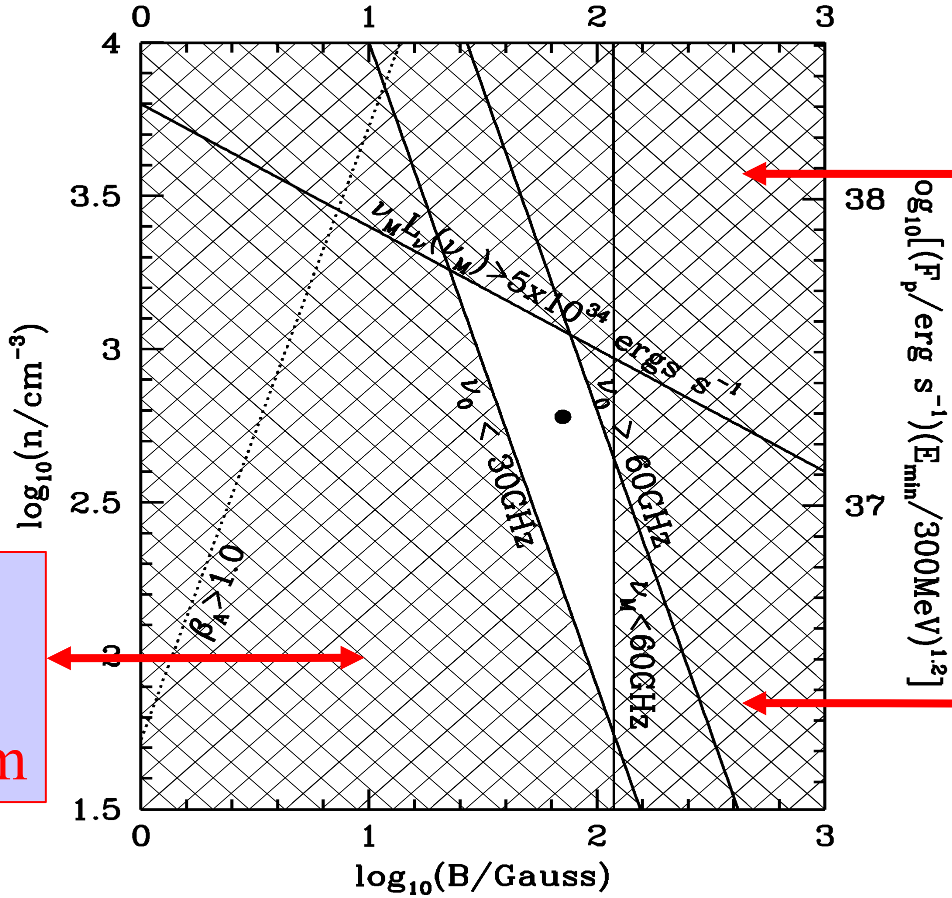
“Quiescent” Electron Emissions



Stochastic Particle Acceleration



Constraints on Parameters



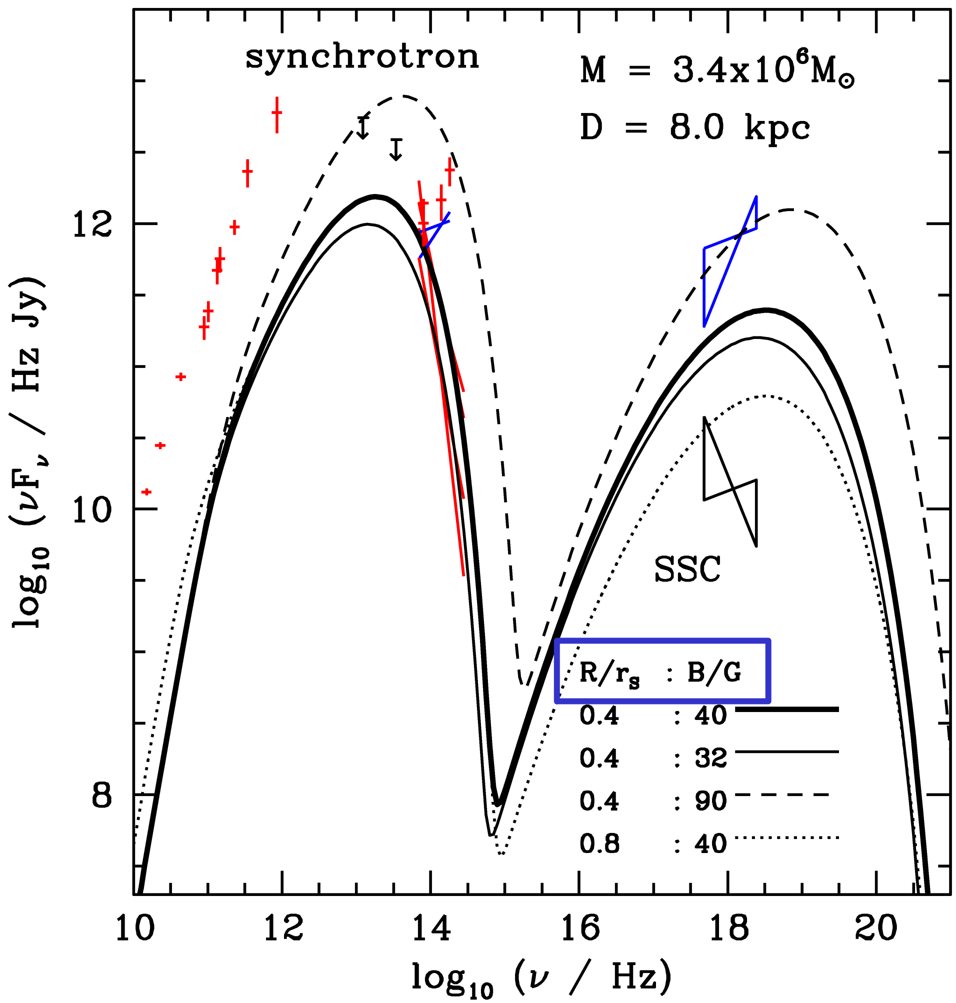
Source is too bright in the radio band.

Source is optically thin at 7mm

Cooling is too efficient to produce 7 mm emission

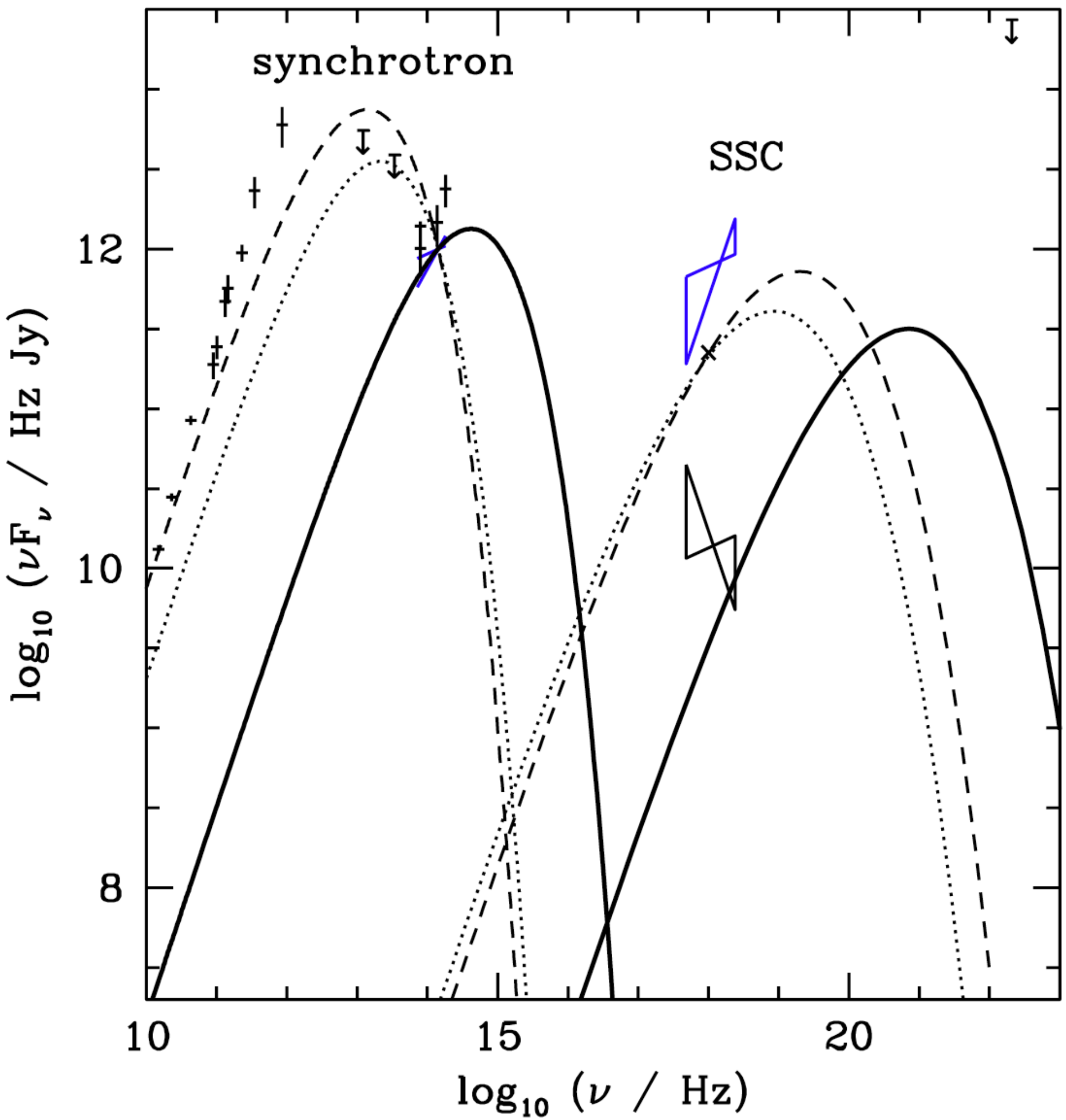
$$\beta_A \equiv \frac{v_A}{c} = 7.3 \left(\frac{B}{1\mathrm{G}}\right) \left(\frac{n}{1\mathrm{cm}^{-3}}\right)^{-1/2} .$$

Emission Processes During Flares



Thermal Synchrotron and SSC:
Four Parameters
$\mathcal{N}$=3.8x10[42]
$k_BT=75m_ec^2$

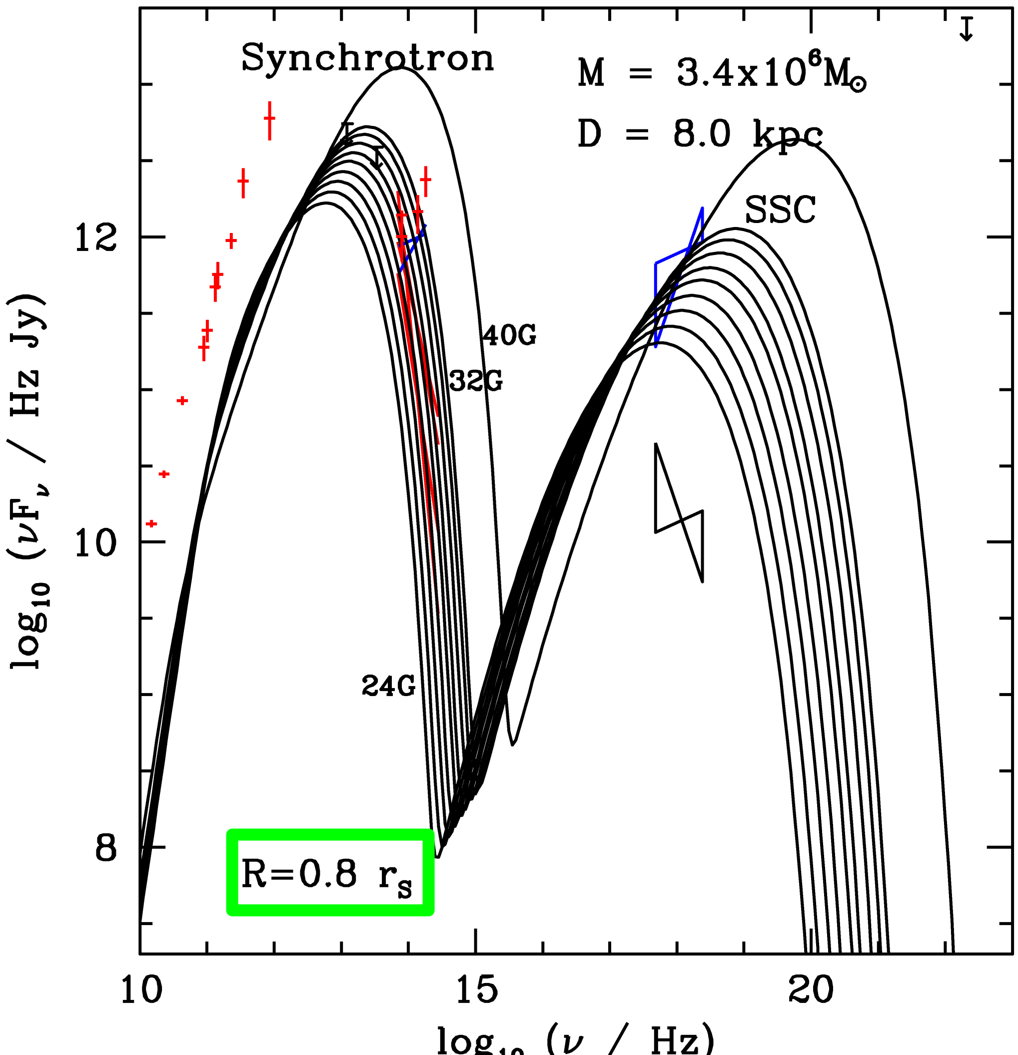
Liu et al. 2006


synchrotron
SSC
12
10
8
10
15
20
log10 (νFν / Hz Jy)
log10 (ν / Hz)

Dependence on B

$$\frac{C_1}{f_{\rm turb}} = 0.72$$

$$C_2 C_1^2 = 0.72$$



Dependence on B

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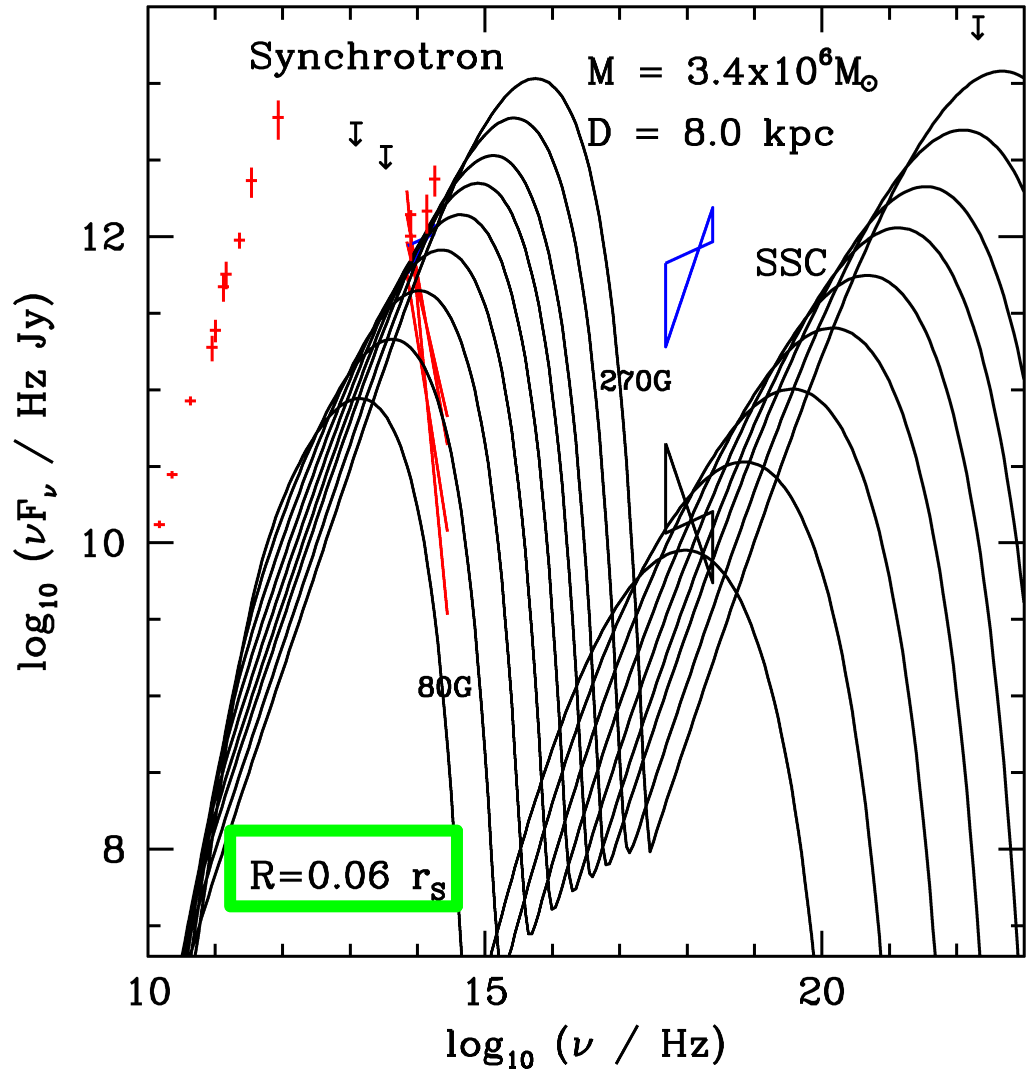
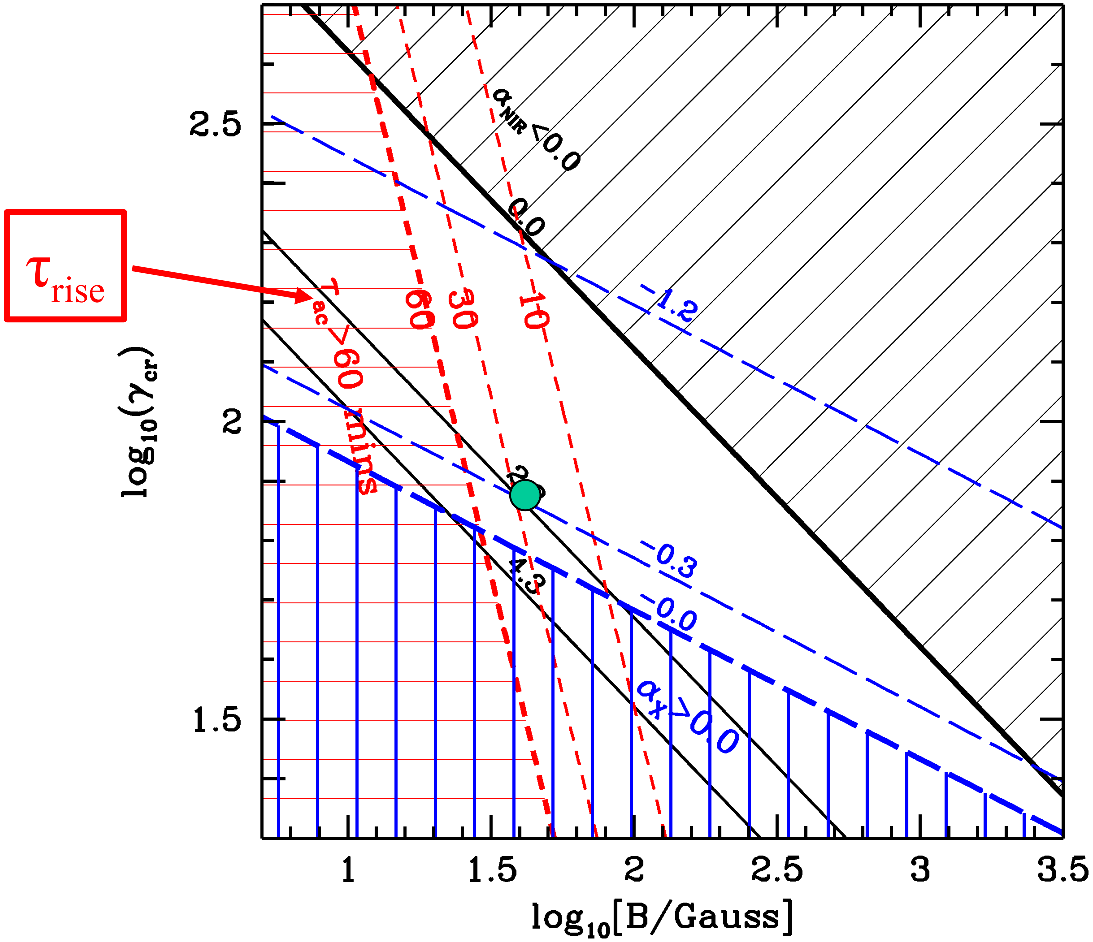


Constraining T & B with NIR and X-ray Spectra and flare rise time



Conclusions

Combination of

stochastic acceleration, MHD simulations,

and observations over a broad range

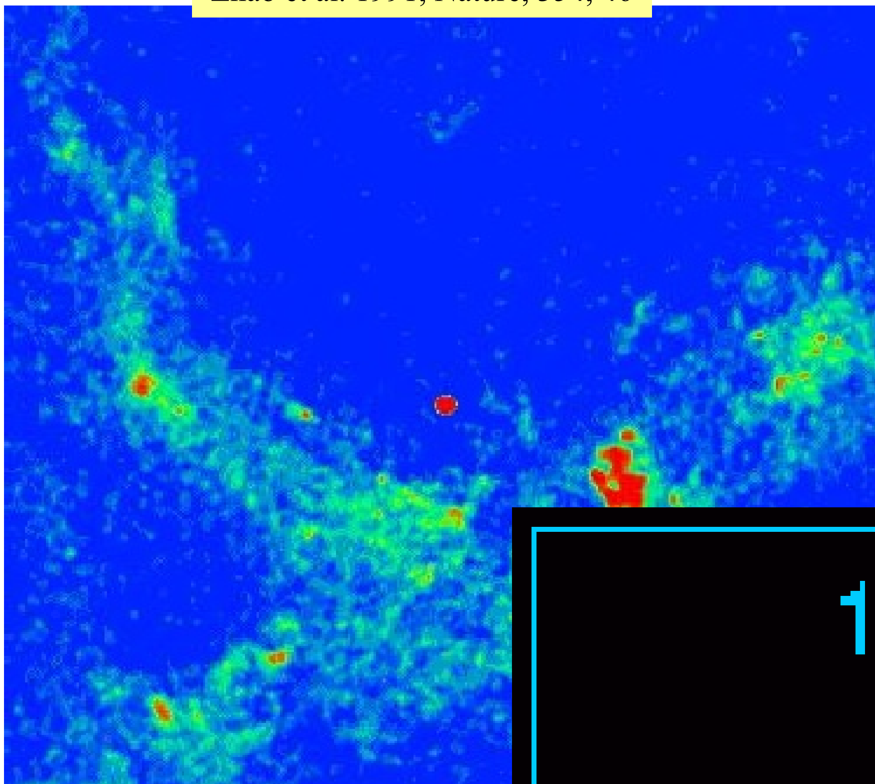
can be used to detect the properties of the black hole and its accretion flows.

SUMMARY

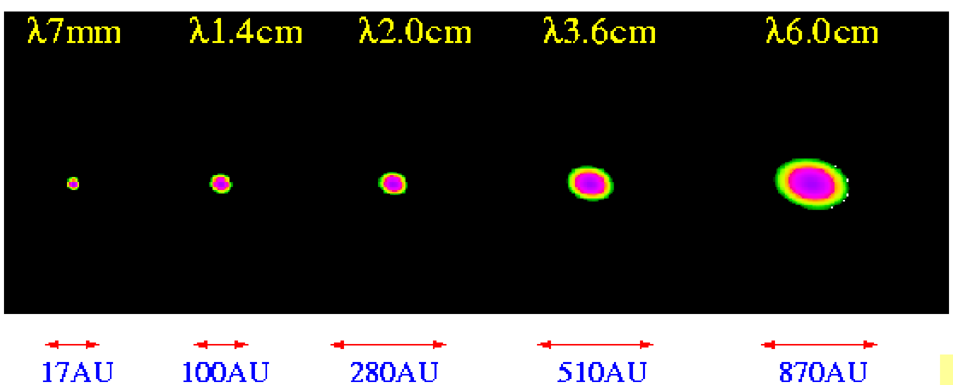
TURBULENCE AND STOCHASTIC ACCELERATION CAN PLAY IMPORTANT ROLES IN MANY ASTROPHYSICAL SOURCES

VLA

Zhao et al. 1991, Nature, 354, 46

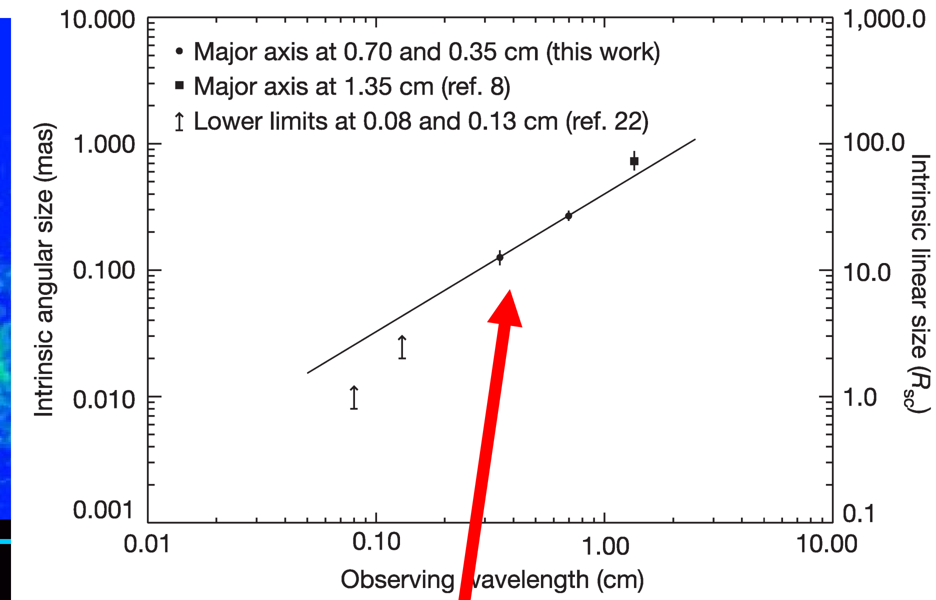


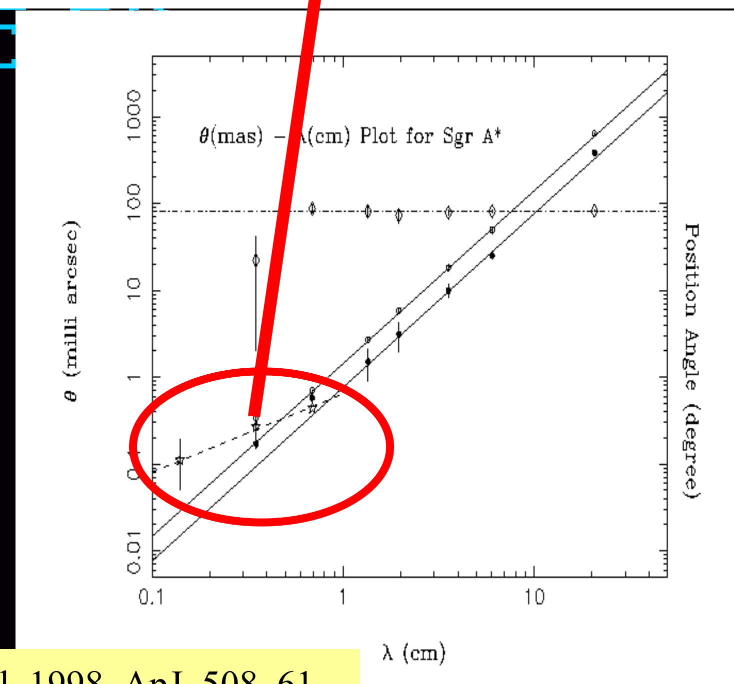
2 cm radio image of the central tw



1995

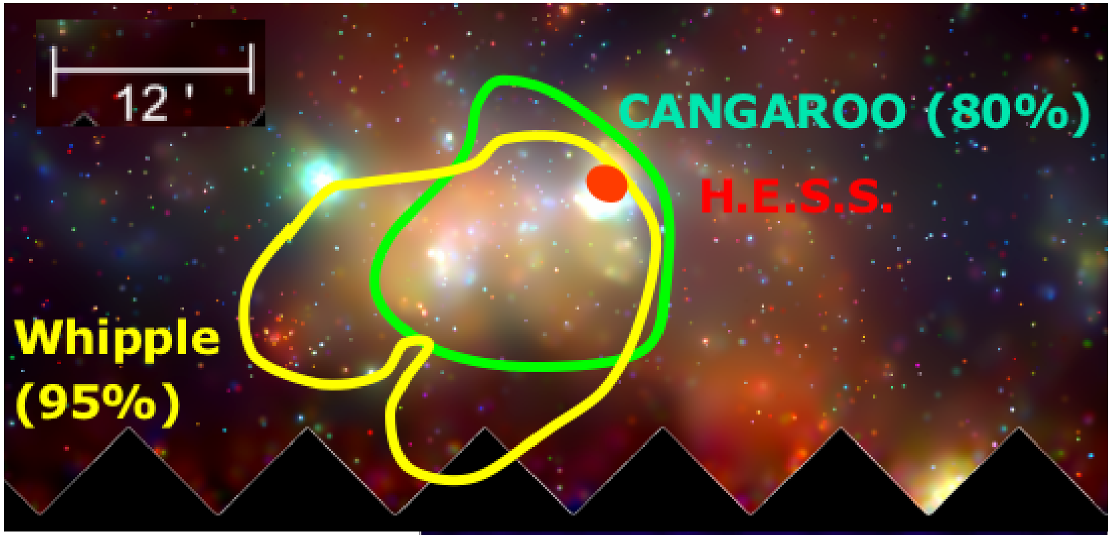
Shen et al. 2005, Nature, 438, 62

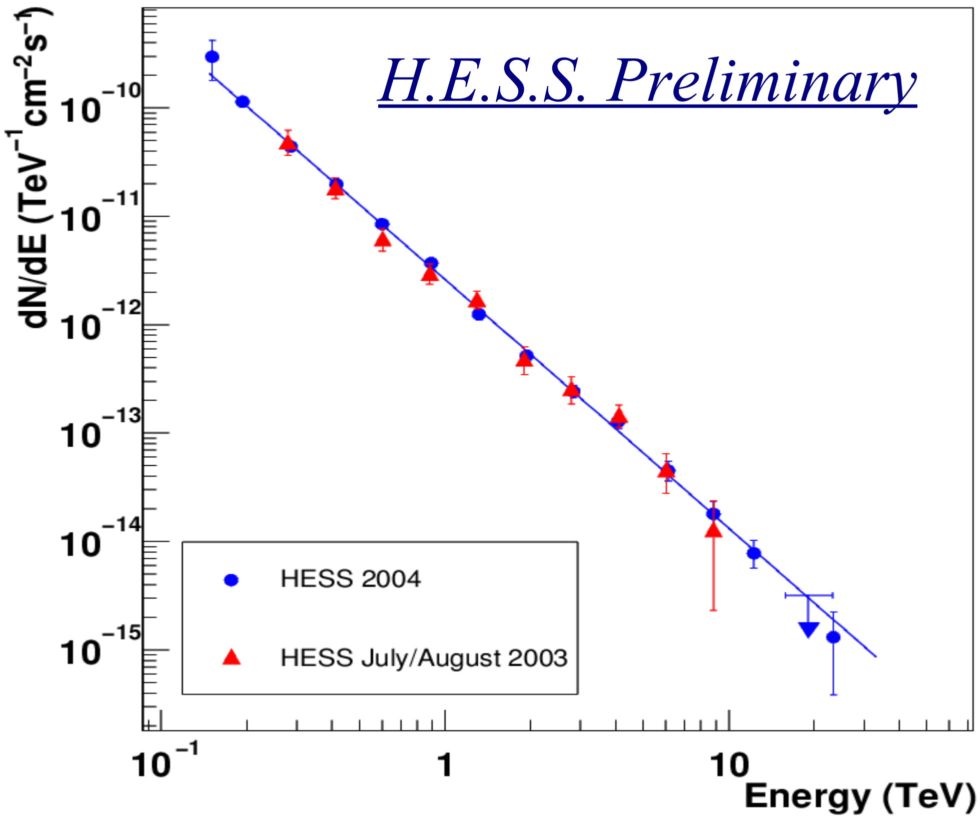


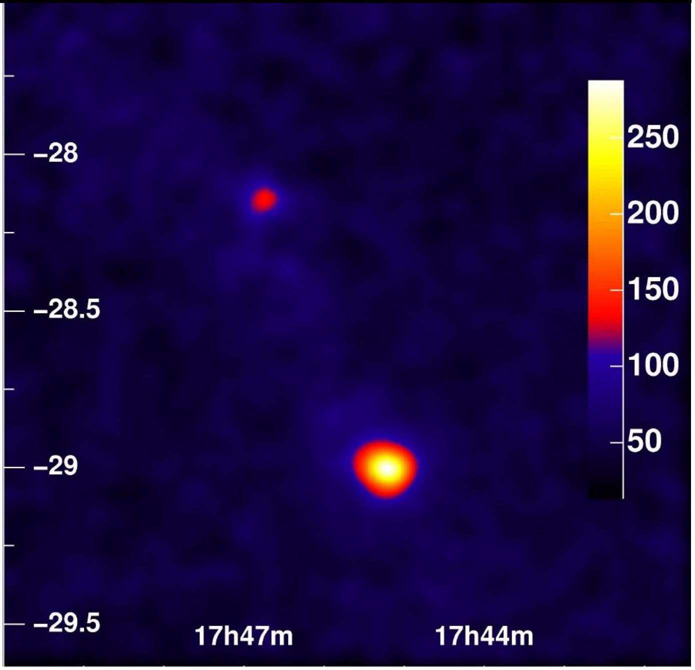


Lo et al. 1998, ApJ, 508, 61

HESS



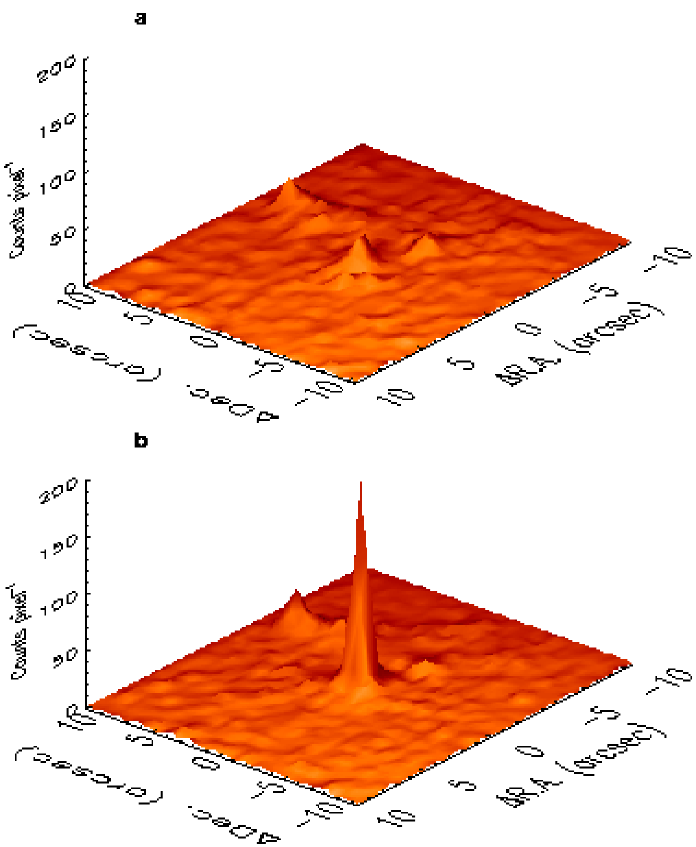


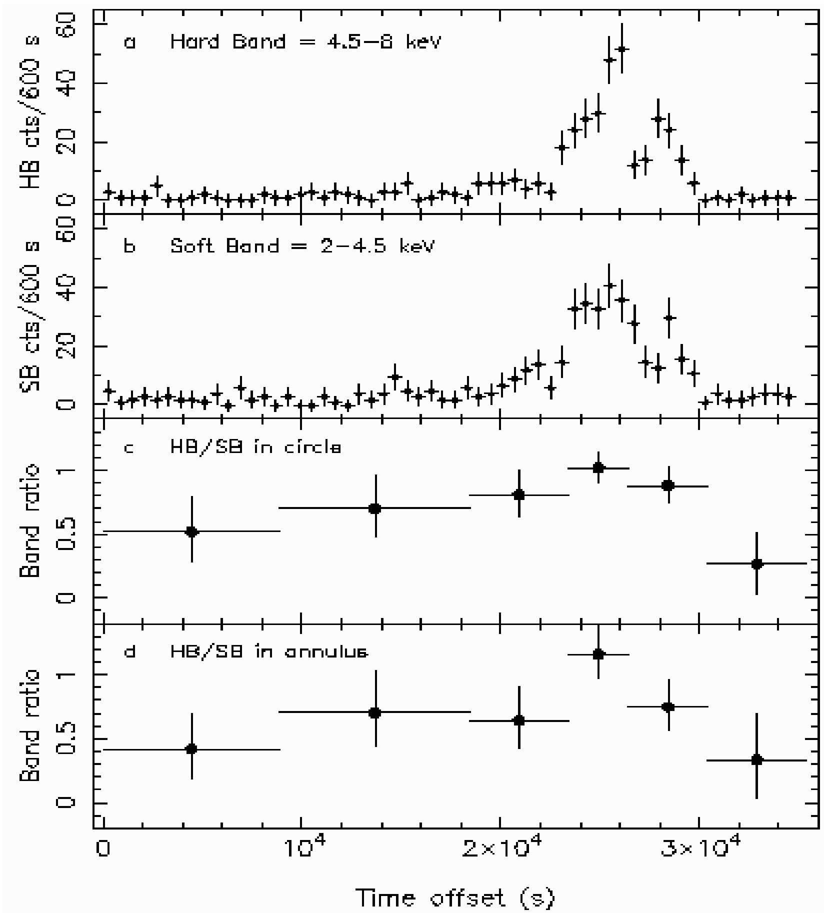


HESS Collaboration 2004

X-ray Flares from Sgr A*

(Baganoff et al. 2001)



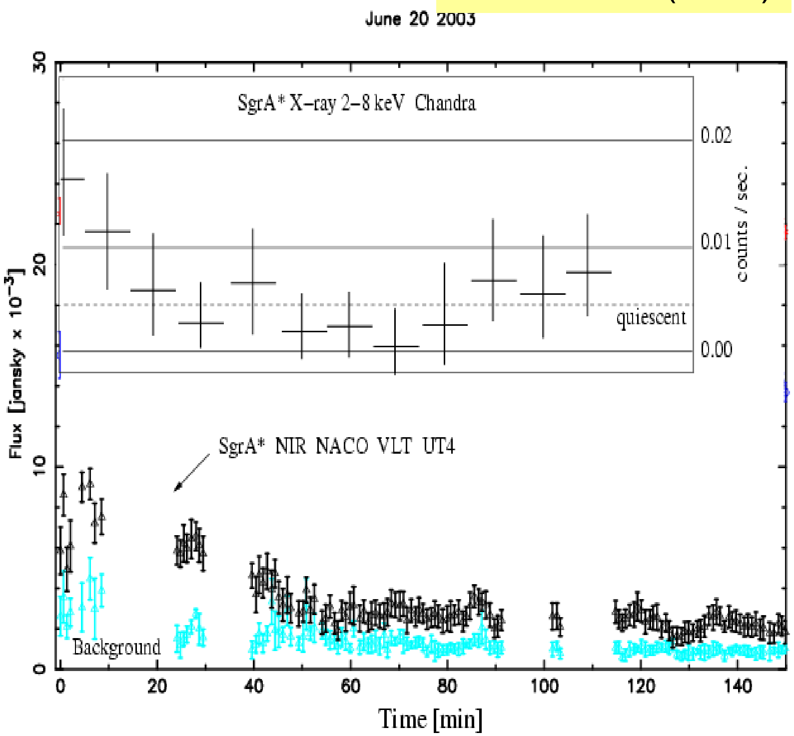


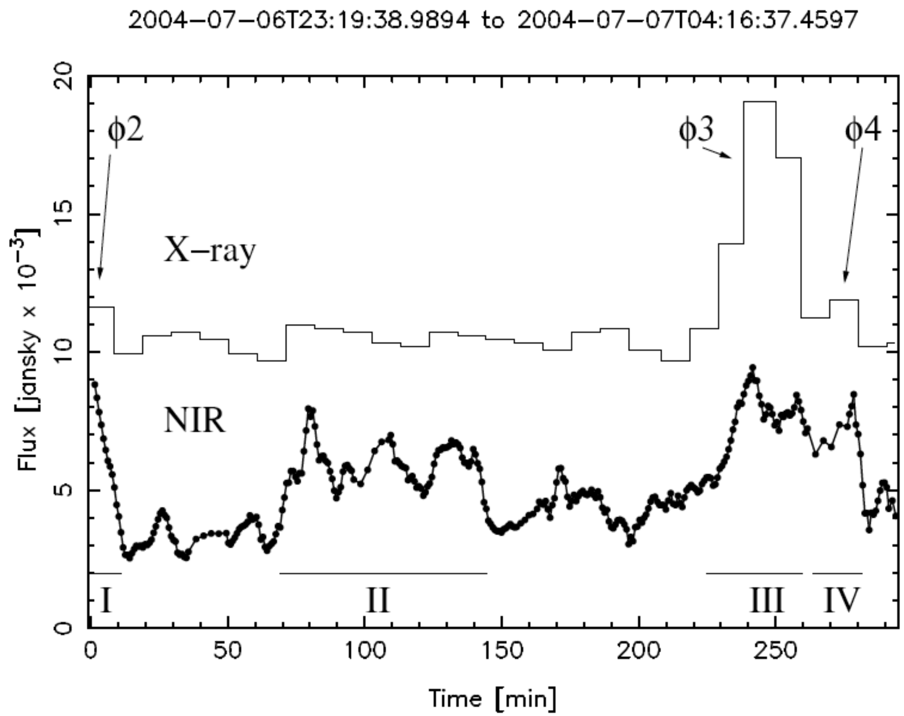
In flare-state, Sgr A*'s X-ray luminosity can increase by more than one order of magnitude.

The X-ray flare lasted for a few hours. Significant variation in flux was seen over a 10 minute interval.

Sgr A* 19-20 June 2003 – NIR/X-ray Flare

Eckart et al. (2004)





Baganoff 2005

$L_x \sim 6\times10^{33}$ erg s^{-1}

$L_{nir} \sim 5\times10^{34}$ erg s^{-1}