Statistical methods for variability studies

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WE-Heraeus Seminar
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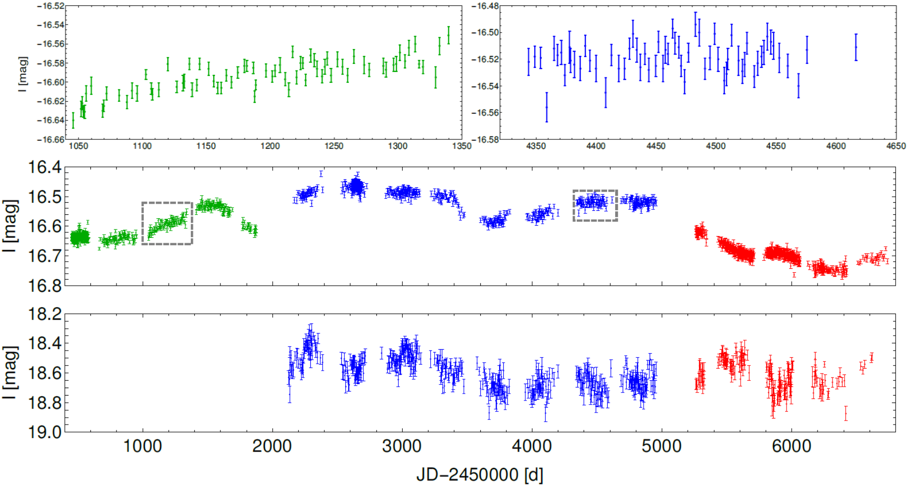
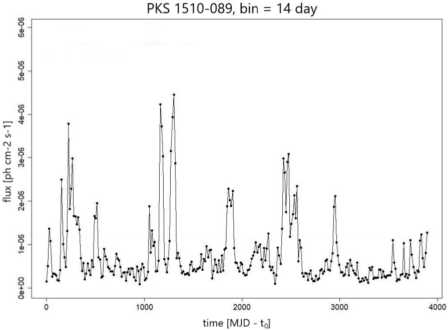
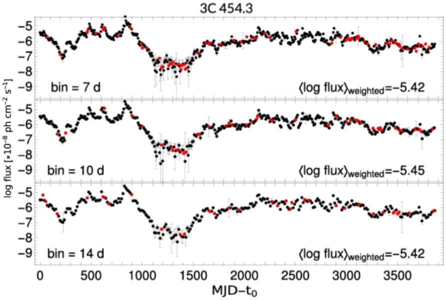
Optical (OGLE) light curves of FSRQ and BL Lac candidates behind Magellanic Clouds:



Żywucka et al. (2018, 2020)

Gamma-ray light curves of Fermi-LAT blazars:





Tarnopolski et al. (2020)

Power spectral density (PSD)

1. power law (PL):

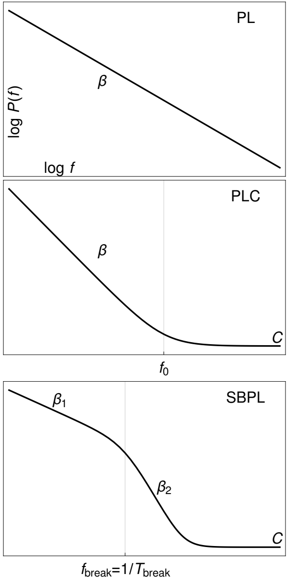
$$P(f) = \frac{P_{\rm norm}}{f^{\beta}}$$

2. PL plus Poisson noise (PLC):

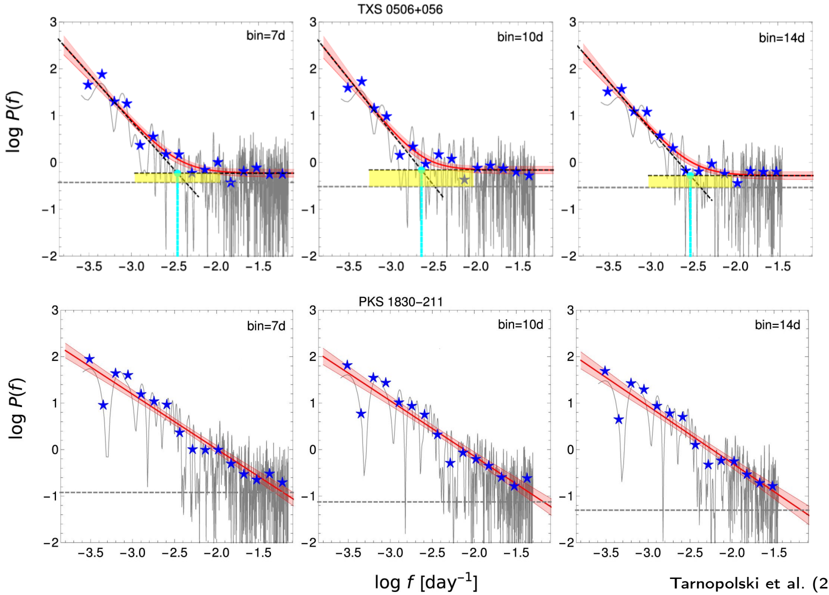
$$P(f) = \frac{P_{\rm norm}}{f^{\beta}} + C$$

3. smoothly broken PL (SBPL) plus Poisson noise:

$$P(f) = \frac{P_{\rm norm} f^{-\beta_1}}{1 + \left(\frac{f}{f_{\rm break}}\right)^{\beta_2 - \beta_1}} + C$$

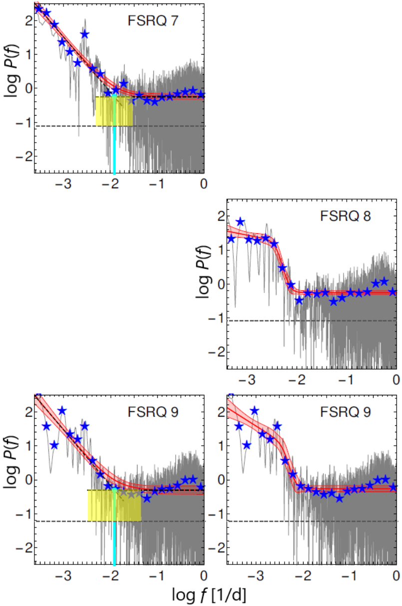


Power spectral density (PSD) — Fermi-LAT



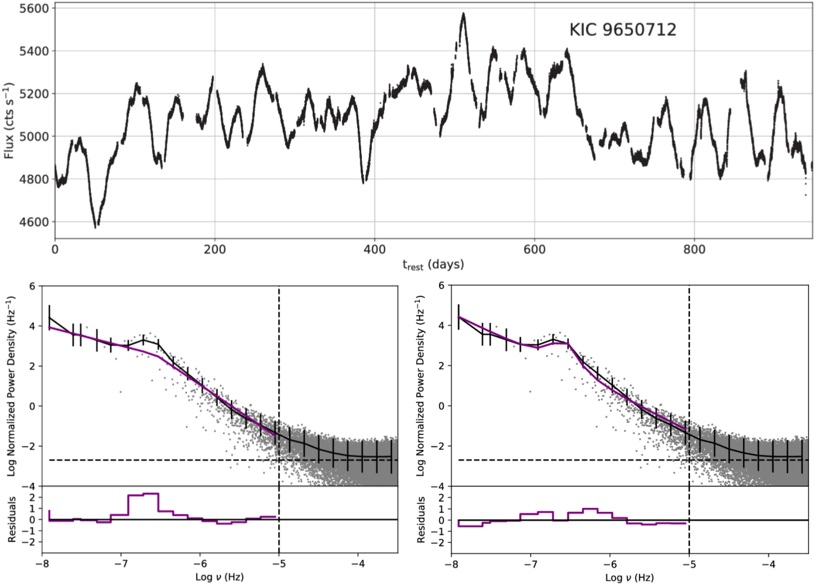
Tarnopolski et al. (2020)

Power spectral density (PSD) — OGLE



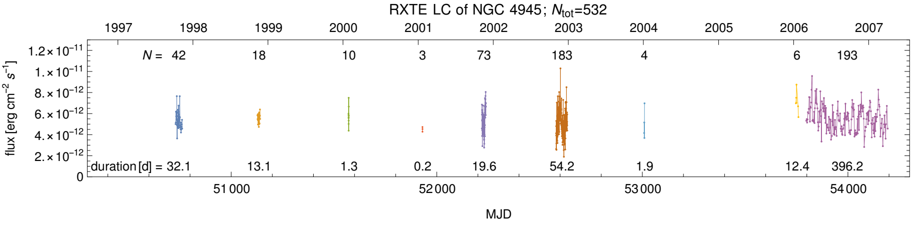
Żywucka et al. (2020)

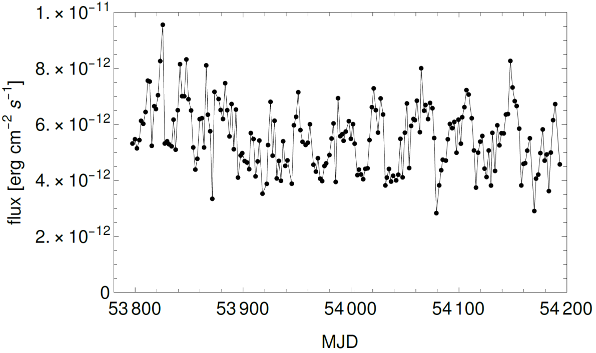
Quasiperiodic oscillations (QPOs) — Kepler



K.L. Smith et al. (2018)

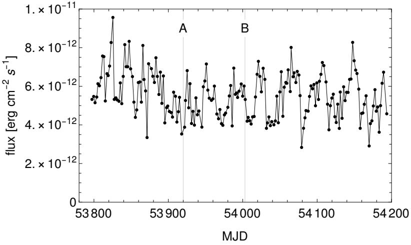
Quasiperiodic oscillations (QPOs) — RXTE

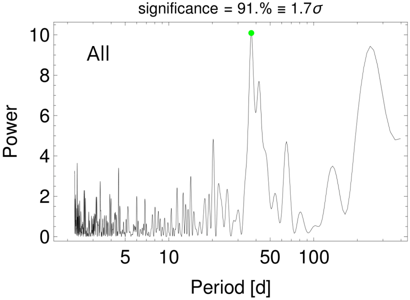




inspired by E. Smith et al. (2020)

Quasiperiodic oscillations (QPOs) — RXTE





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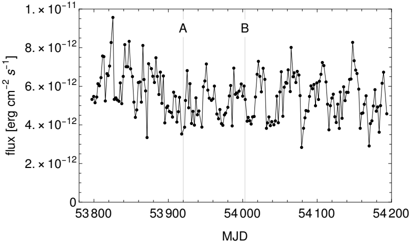

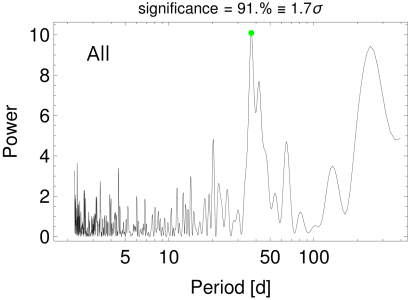

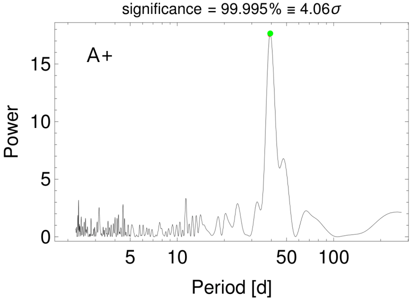

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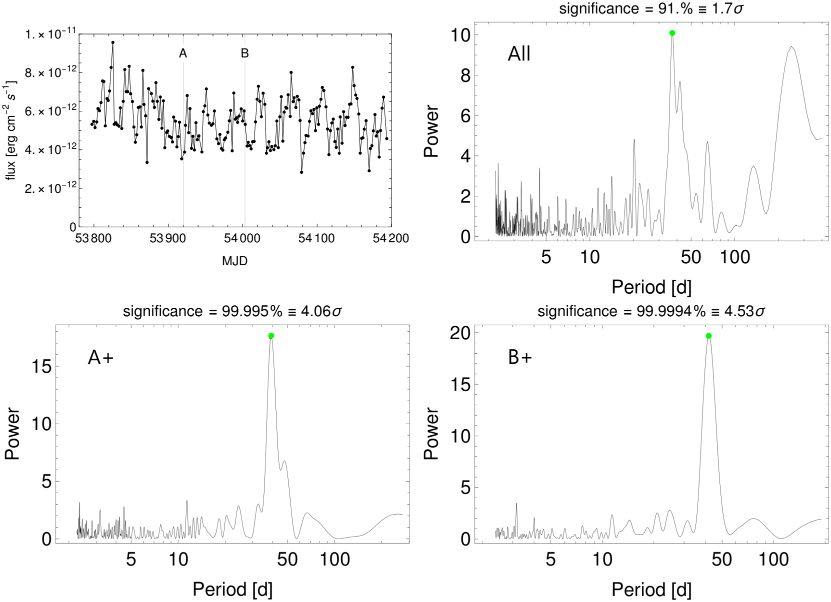


significance = 91.% ≡ 1.7σ

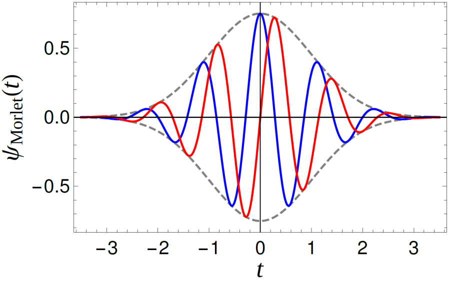
significance = 99.995% ≡ 4.06σ

significance = 99.9994% ≡ 4.53σ

MT & V. Marchenko (unpublished)

Quasiperiodic oscillations (QPOs) via wavelets

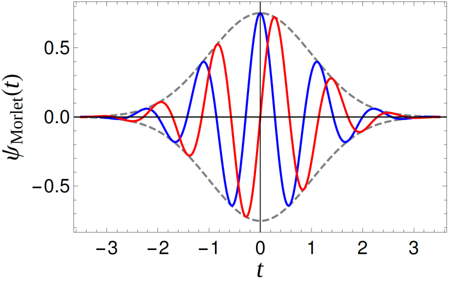
A (mother) wavelet $\psi(t)$ is a short, temporally and spectrally localized oscillation.



Quasiperiodic oscillations (QPOs) via wavelets

A (mother) wavelet $\psi(t)$ is a short, temporally and spectrally localized oscillation. Child wavelets form a basis (l—translation, s—scale):

$$\psi_{s,l}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-l}{s}\right)$$



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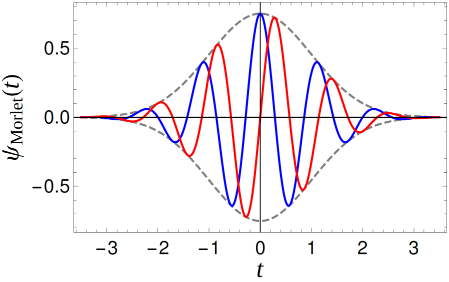
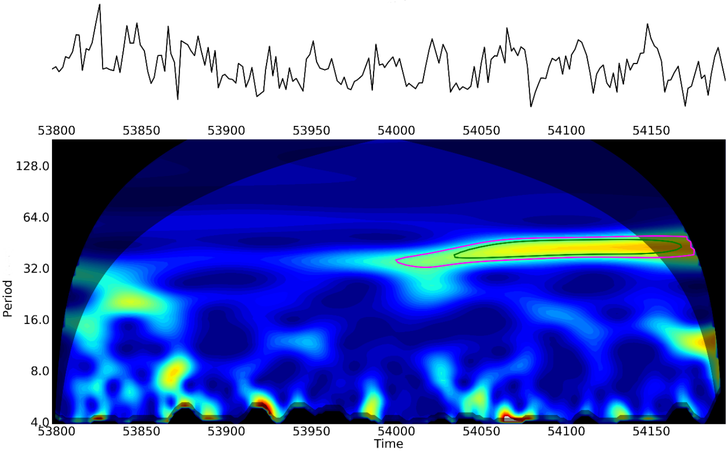


$$x(t) = \sum_{s,l} W(s,l)\psi_{s,l}(t)$$

$$W(s,l) = \int_t x(t)\psi^*_{s,l}(t)\mathrm{d}t$$

$$P_{\mathrm{wav}}(s,l) = |W(s,l)|^2$$

Quasiperiodic oscillations (QPOs) — RXTE

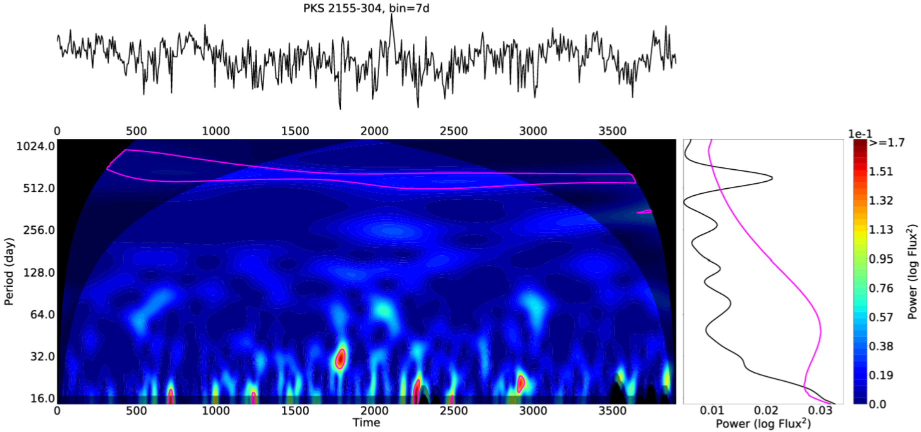


MT & V. Marchenko; using WAVEPAL (Lenoir & Crucifix 2018)

Statistical significance via CARMA models

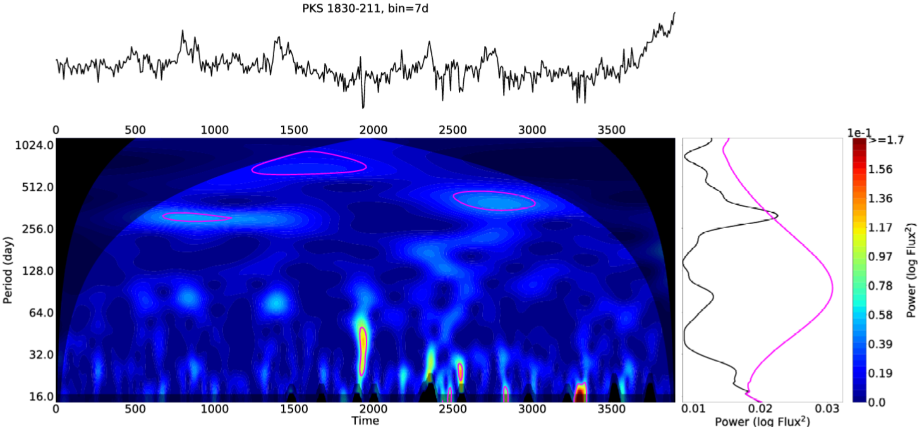
(cf. Sz. Kozłowski's talk & Kelly et al. 2014)

Quasiperiodic oscillations (QPOs) — Fermi-LAT



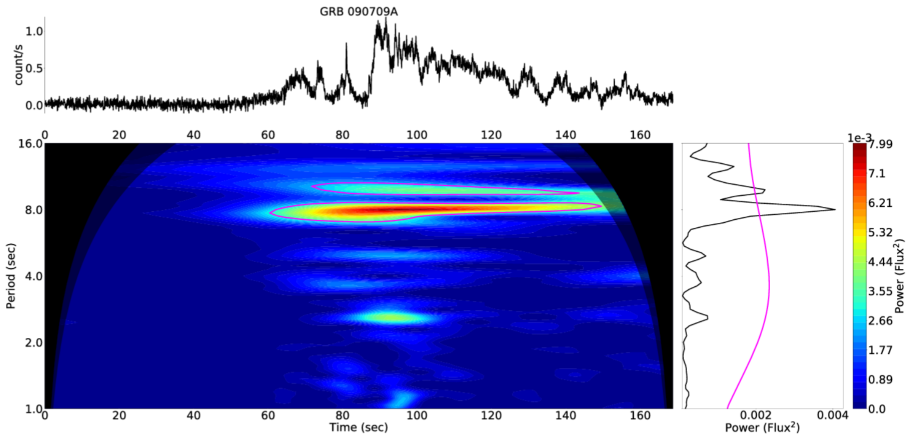
Tarnopolski et al. (2020)

Quasiperiodic oscillations (QPOs) — Fermi-LAT



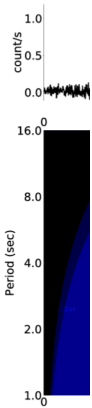
Tarnopolski et al. (2020)

Quasiperiodic oscillations (QPOs) — Swift-BAT GRBs



Tarnopolski & Marchenko (2021)

Quasipe wift-BAT GRBs



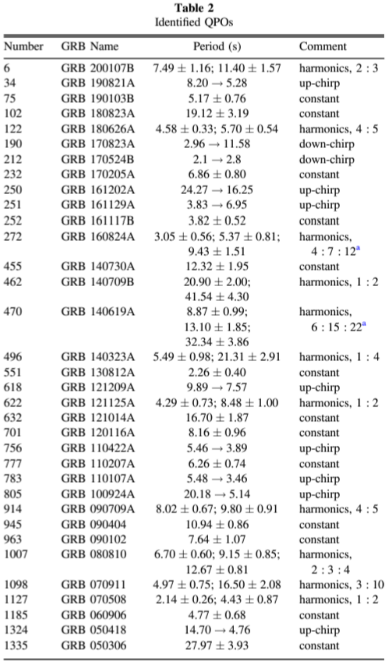
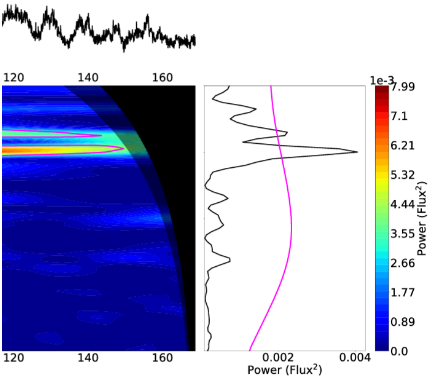


Table 2
Identified QPOs

Number	GRB Name	Period (s)	Comment
6	GRB 200107B	7.49 ± 1.16; 11.40 ± 1.57	harmonics, 2 : 3
34	GRB 190821A	8.20 → 5.28	up-chirp
75	GRB 190103B	5.17 ± 0.76	constant
102	GRB 180823A	19.12 ± 3.19	constant
122	GRB 180626A	4.58 ± 0.33; 5.70 ± 0.54	harmonics, 4 : 5
190	GRB 170823A	2.96 → 11.58	down-chirp
212	GRB 170524B	2.1 → 2.8	down-chirp
232	GRB 170205A	6.86 ± 0.80	constant
250	GRB 161202A	24.27 → 16.25	up-chirp
251	GRB 161129A	3.83 → 6.95	up-chirp
252	GRB 161117B	3.82 ± 0.52	constant
272	GRB 160824A	3.05 ± 0.56; 5.37 ± 0.81; 9.43 ± 1.51	harmonics, 4 : 7 : 12[a]
455	GRB 140730A	12.32 ± 1.95	constant
462	GRB 140709B	20.90 ± 2.00; 41.54 ± 4.30	harmonics, 1 : 2
470	GRB 140619A	8.87 ± 0.99; 13.10 ± 1.85; 32.34 ± 3.86	harmonics, 6 : 15 : 22[a]
496	GRB 140323A	5.49 ± 0.98; 21.31 ± 2.91	harmonics, 1 : 4
551	GRB 130812A	2.26 ± 0.40	constant
618	GRB 121209A	9.89 → 7.57	up-chirp
622	GRB 121125A	4.29 ± 0.73; 8.48 ± 1.00	harmonics, 1 : 2
632	GRB 121014A	16.70 ± 1.87	constant
701	GRB 120116A	8.16 ± 0.96	constant
756	GRB 110422A	5.46 → 3.89	up-chirp
777	GRB 110207A	6.26 ± 0.74	constant
783	GRB 110107A	5.48 → 3.46	up-chirp
805	GRB 100924A	20.18 → 5.14	up-chirp
914	GRB 090709A	8.02 ± 0.67; 9.80 ± 0.91	harmonics, 4 : 5
945	GRB 090404	10.94 ± 0.86	constant
963	GRB 090102	7.64 ± 1.07	constant
1007	GRB 080810	6.70 ± 0.60; 9.15 ± 0.85; 12.67 ± 0.81	harmonics, 2 : 3 : 4
1098	GRB 070911	4.97 ± 0.75; 16.50 ± 2.08	harmonics, 3 : 10
1127	GRB 070508	2.14 ± 0.26; 4.43 ± 0.87	harmonics, 1 : 2
1185	GRB 060906	4.77 ± 0.68	constant
1324	GRB 050418	14.70 → 4.76	up-chirp
1335	GRB 050306	27.97 ± 3.93	constant



Tarnopolski & Marchenko (2021)

Caveat emptor

Significance levels:

per cent	sigma
68%	1σ
90%	1.64σ
95%	1.96σ
99%	2.58σ
99.73%	3σ
99.9937%	4σ
99.9999%	5σ

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Oops-Leon

- 6 GeV/c^2 particle; significance 98% ($\equiv 2.33\sigma$); denoted *Upsilon* (Υ).
- More data ⇒ spurious discovery.
- A 9.5 GeV/c^2 particle discovered soon after at a 5σ level—reused the name *Upsilon*.

Hom et al., PRL, **36**, 1236 (1976); **39**, 252 (1977)

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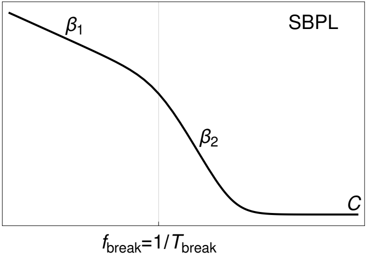
Black hole mass estimates



$$T_B = 2\pi(r_{\rm ISCO}^{3/2} + a_\star)(1+z)\frac{GM_{\rm BH}}{c^3}$$
$$= 0.359\, m_9(1+z) f(a_\star)\,{\rm day}$$

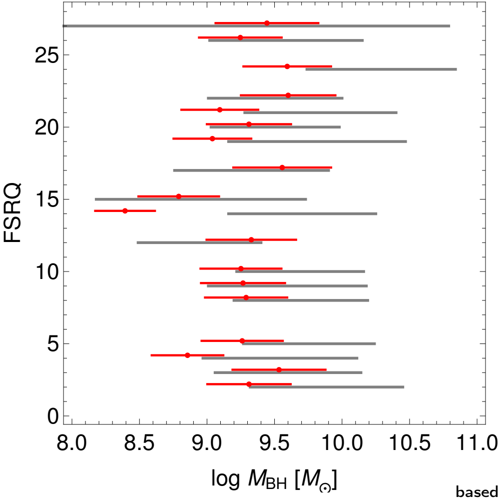
$$T_{\rm break} \sim t_{\rm th} \sim \alpha^{-1} t_{\rm K}$$

$$T_B \sim t_{\rm K} \sim \alpha T_{\rm break}$$

Black hole mass estimates

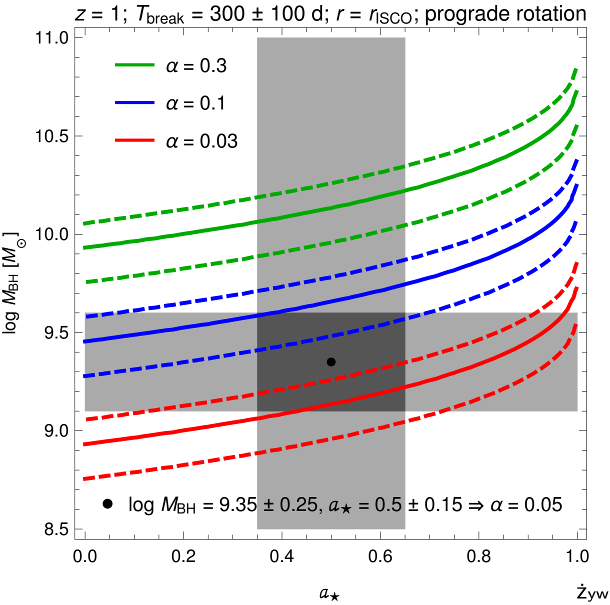
$$\log\frac{T_{\rm break}}{1\,{\rm day}} = A\log\frac{M_{\rm BH}}{10^6 M_\odot} - B\log\frac{L_{\rm bol}}{10^{44}\,{\rm erg\,s^{-1}}} + C$$

McHardy et al. (2006)



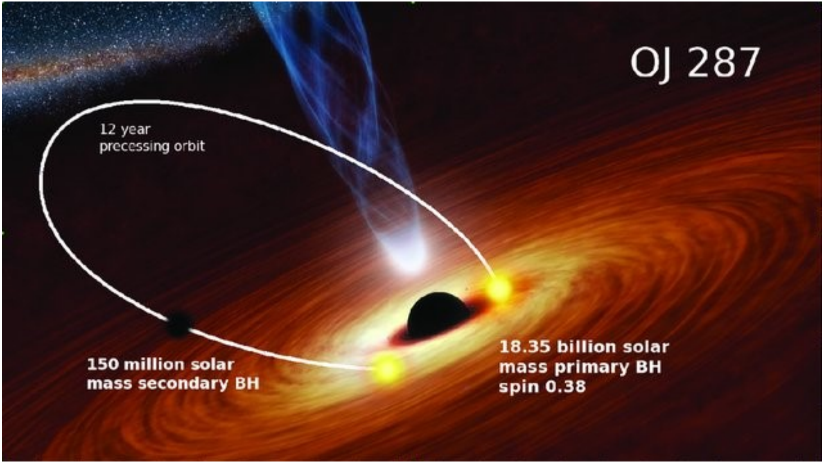
based on Żywucka et al. (2020)

Viscosity estimates



Żywucka et al. (2020)

OJ 287 — binary SMBH



S. Zoła & NASA/JPL

Binary SMBHs?

All that's periodic is not a binary black hole
(Shakespeare, travestied, 1596)

- It has been speculated that even 10% of blazars can be a binary SMBH system
- They would contribute to the nHz GW background (cf. D. Champion's talk)
- Comparison with the pulsar timing array implies that binary SMBHs can constitute $\lesssim 0.1\%$ blazars (Holgado et al. 2018)

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QPOs — what else can cause them?

- Lense-Thirring eff. → disk precession — $t_{LT} = \frac{8\pi GM}{c^3 a_\star}\left(\frac{r}{R_S}\right)^3 \sim 0.1 \div 10\,\mathrm{yrs}$
- Warped accretion disks
- Helical jet/magnetic fields; helical motion within the jet; twisting filaments in the jet etc.
- Perturbations in the accretion disk (matter or magnetic field densities, etc.) in the vicinity of the SMBH can propagate into the jet — however, e.g.:
  - variations in $\dot{m}$ propagate through the disk with a time scale $t_d \simeq \frac{1}{2\pi\alpha}\left(\frac{r}{H}\right)^2 t_K \sim 10^{2\div 3}\,\mathrm{yrs}$
  - but $t_{th} \simeq \frac{1}{2\pi\alpha} t_K \sim 0.1 \div 10\,\mathrm{yrs}$ (but $a_\star \neq 0$ in general)
- Jet's precession, even a slight one, changes the viewing angle — strong dependence of the Doppler factor δ on the viewing angle → flux changes $F_\nu = \delta^3 F'_\nu$ (also QPOs in polarization)
- Gravitational lensing (cf. D. Król's talk)
- Multitude of other scenarios — possibly a combination of several of them

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- Gravitational lensing (cf. D. Król's talk)
- Multitude of other scenarios — possibly a combination of several of them

QPOs — what else can cause them?

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- Helical jet/magnetic fields; helical motion within the jet; twisting filaments in the jet etc.
- Perturbations in the accretion disk (matter or magnetic field densities, etc.) in the vicinity of the SMBH can propagate into the jet — however, e.g.:
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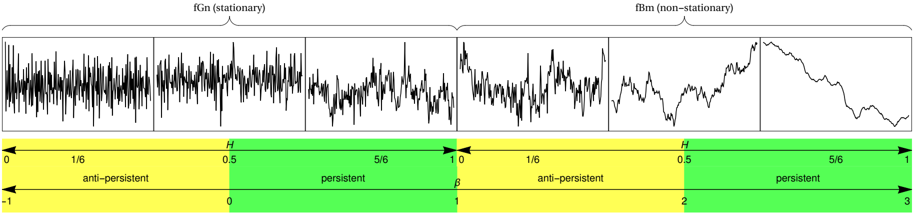
Hurst exponents

$$x(t) \doteq \lambda^{-H} x(\lambda t) \qquad \rho_k \propto |k|^{-\delta} \equiv |k|^{-(2-2H)}$$



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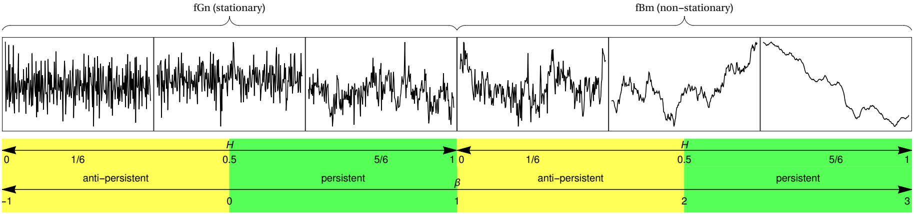
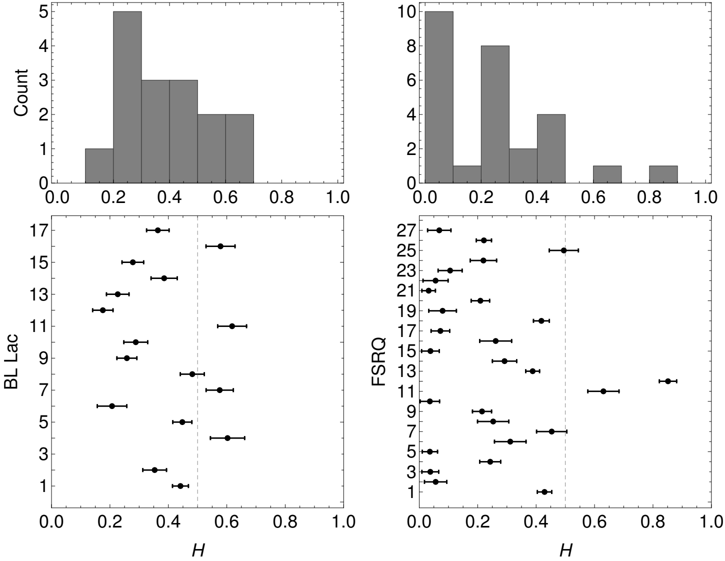


The properties of H:

1. $0 < H < 1$,
2. $H = 1/2$ for an uncorrelated process,
3. $H > 1/2$ for a persistent (long-term memory, correlated) process,
4. $H < 1/2$ for an anti-persistent (short-term memory, anti-correlated) process.

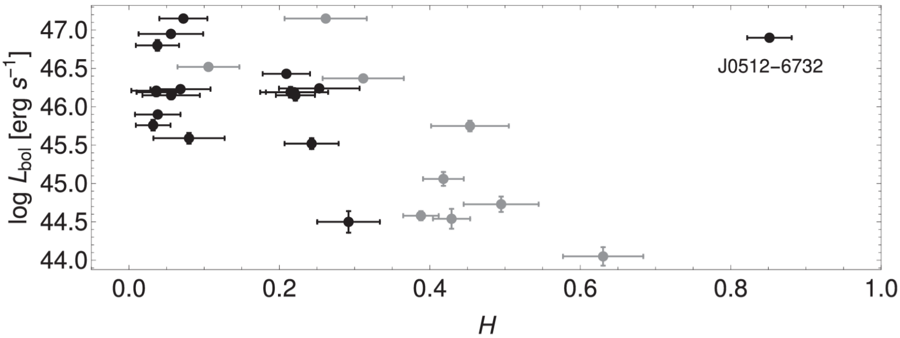
Tarnopolski et al. (2020), Żywucka et al. (2020)

Hurst exponents — OGLE



Żywucka et al. (2020)

Hurst exponents — OGLE

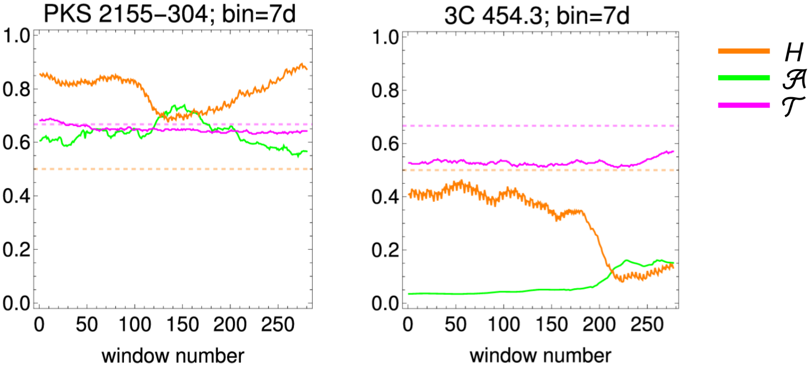


Black – SBPL; gray – PL.

$r = -0.7$

Żywucka et al. (2020)

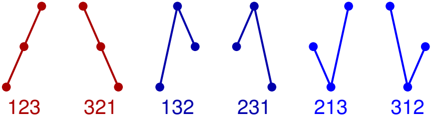
Hurst exponents — Fermi-LAT



Tarnopolski et al. (2020)

$\mathcal{A}-\mathcal{T}$ plane

1. Fraction of turning points, $\mathcal{T}$:



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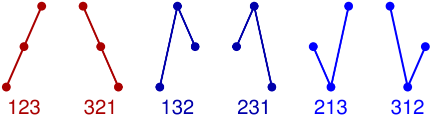
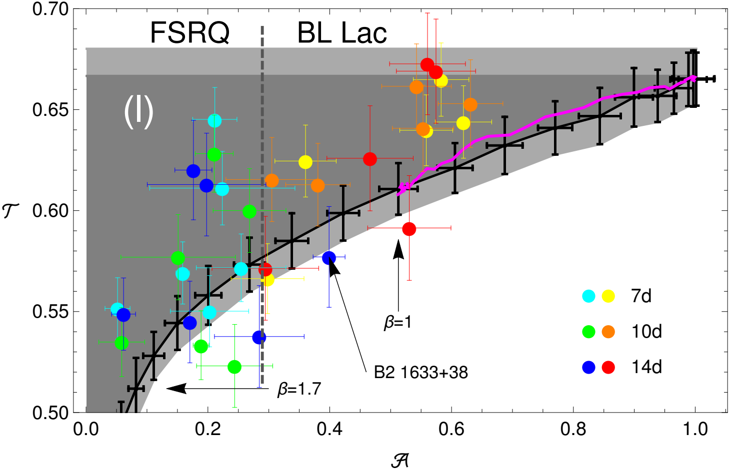


2. Abbe value:

$$\mathcal{A} = \frac{\frac{1}{N-1}\sum\limits_{k=1}^{N-1}(x_{k+1}-x_k)^2}{\frac{2}{N}\sum\limits_{k=1}^{N}(x_k-\bar{x})^2} = \frac{1}{2}\frac{\mathrm{var}\,(dX)}{\mathrm{var}\,(X)}$$

Tarnopolski (2016, 2019)

$\mathcal{A} - \mathcal{T}$ plane — Fermi-LAT



Tarnopolski et al. (2020)

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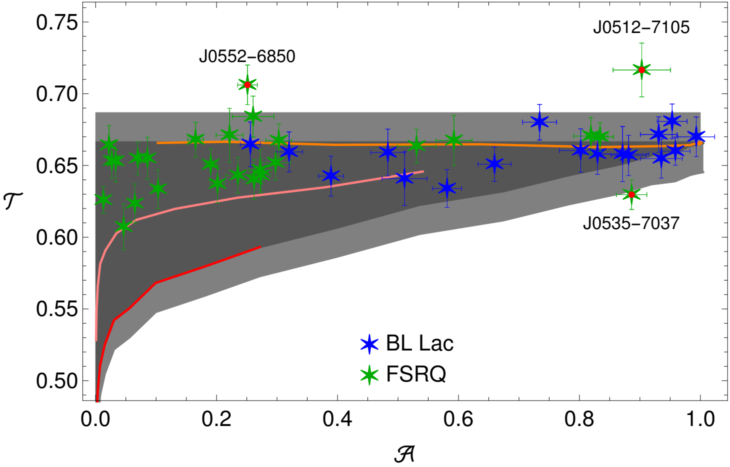

$\langle\mathcal{A}\rangle_{\rm FSRQ} = 0.29 \pm 0.05$ and $\langle\mathcal{A}\rangle_{\rm BLLac} = 0.71 \pm 0.06$

Żywucka et al. (2020)

Summary

- Wavelet scalograms, optimized for irregularly spaced data, and with **significance testing — crucial**
- Not every QPO indicates a binary SMBH
- Long, dense, high quality data → **more involved phenomenological stochastic models** — e.g., Hurst exponents capture some fine details about the autocorrelations
- $\mathcal{A}-\mathcal{T}$ plane — classification etc.
- **Connections with physical properties — important**

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☺ Thank you for your attention ☺

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