

# Probing Lorentz Invariance Violations with High Energy Cosmic Particles (Heraeus seminar)

1. Lorentz Symmetry Violation and Propagation
2. Lorentz Symmetry Violation and Air Showers



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DER FORSCHUNG | DER LEHRE | DER BILDUNG

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# Lorentz symmetry violations in the Nucleon Sector

Dispersion relation between energy  $E$ , momentum  $p$ , and mass  $m$  may be modified by non-renormalizable effects at the Planck scale  $M_{Pl}$ ,

$$E^2 - p^2 \simeq m^2 - \xi \frac{p^3}{M_{Pl}} - \zeta \frac{p^4}{M_{Pl}^2} + \dots$$

where most models, e.g. critical string theory, predict  $\xi=0$  for lowest order.

Introducing the standard threshold momentum for pion production,  $N+\gamma \rightarrow N\pi$ ,

$$p_0 = \frac{2m_N m_\pi + m_\pi^2}{4\epsilon}$$

the threshold momentum  $p_{th}$  in the modified theory is given by

$$-\frac{p_0^3}{(m_\pi^2 + 2m_\pi m_N) M_{Pl}} \frac{m_\pi m_N}{(m_\pi + m_N)^2} \left[ 2\xi \left( \frac{p_{th}}{p_0} \right)^3 + 3\zeta \frac{p_0}{M_{Pl}} \left( \frac{p_{th}}{p_0} \right)^4 + \dots \right] + \frac{p_{th}}{p_0} = 1$$

Attention: this assumes standard energy-momentum conservation which is not necessarily the case.

For  $\xi \sim \zeta \sim 1$  this equation has no solution  $\Rightarrow$  No GZK threshold!

For  $\zeta \sim 0, \xi \sim -1$  the threshold is at  $\sim 1$  PeV!

For  $\xi \sim 0, \zeta \sim -1$  the threshold is at  $\sim 1$  EeV!

Confirmation of a normal GZK threshold would imply the following limits:

$|\xi| < 10^{-13}$  for the first-order effects.

$|\zeta| < 10^{-6}$  for the second-order effects.

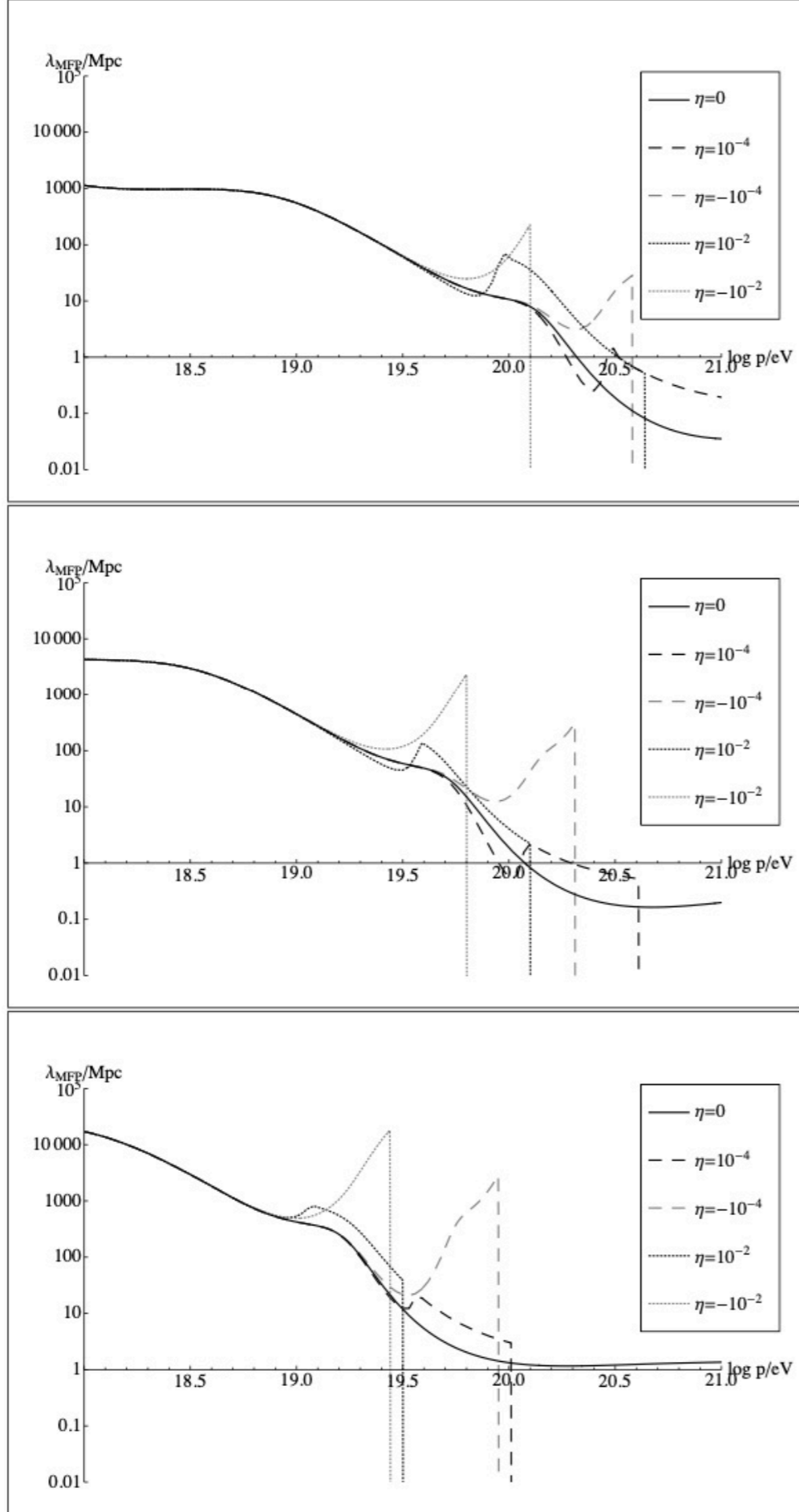
But note that existence of GZK-caused "cut-off" is not sure these days !

Energy-independent (renormalizable) corrections to the maximal speed

$V_{\max} = \lim_{E \rightarrow \infty} \partial E / \partial p = 1 - d$  can be constrained by substituting

$d \rightarrow (\xi/2)(E/M_{\text{pl}}) + (\zeta/2)(E/M_{\text{pl}})^2$ .

# Influence on nuclei mean free path



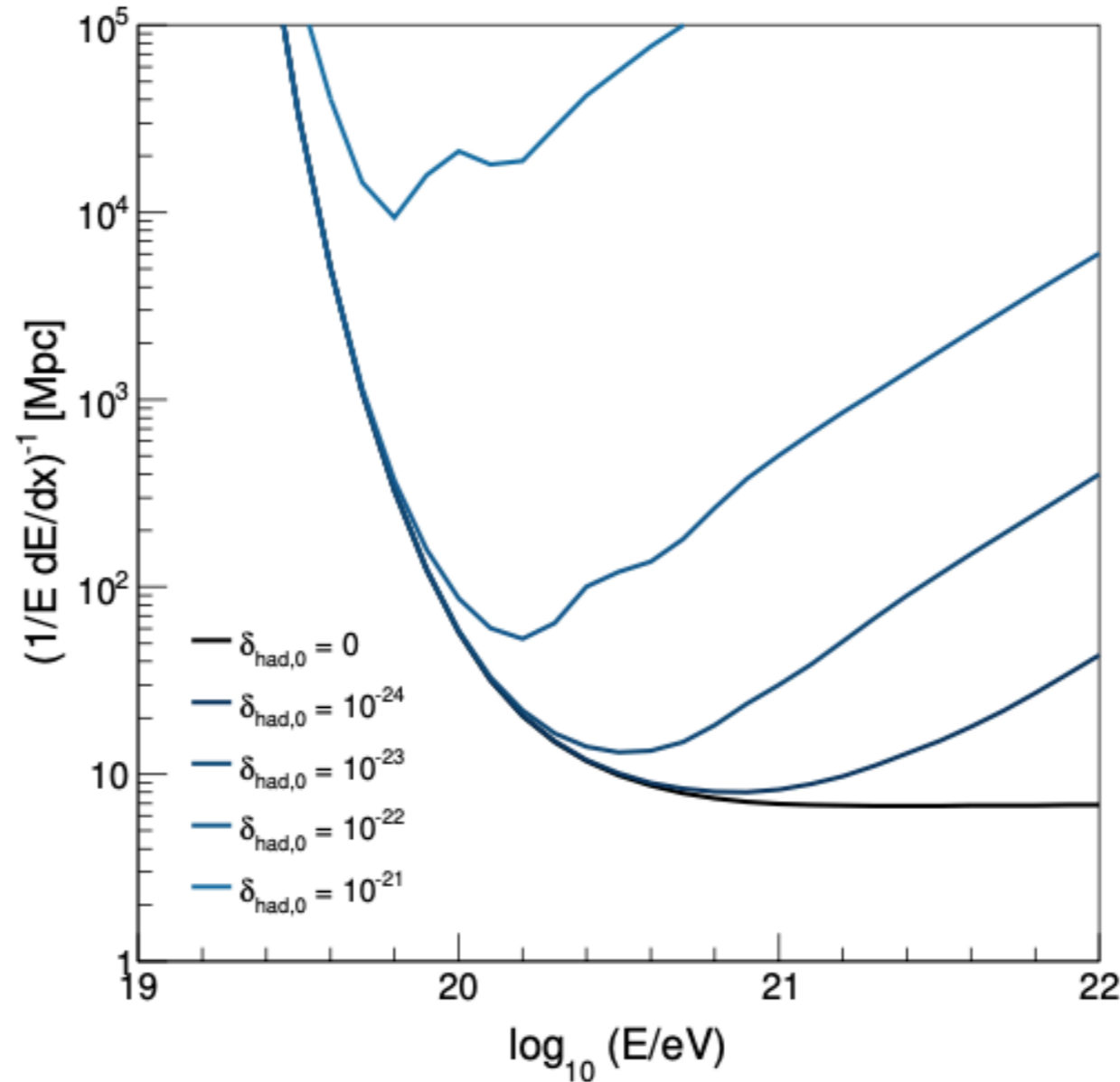
Saveliev, Maccione, Sigl, JCAP 03 (2011) 046

**Figure 3.** The mean free path of photodisintegration in the single parameter model for  $^{56}\text{Fe}$  (top),  $^{16}\text{O}$  (middle) and  $^4\text{He}$  (bottom).

Here the notation

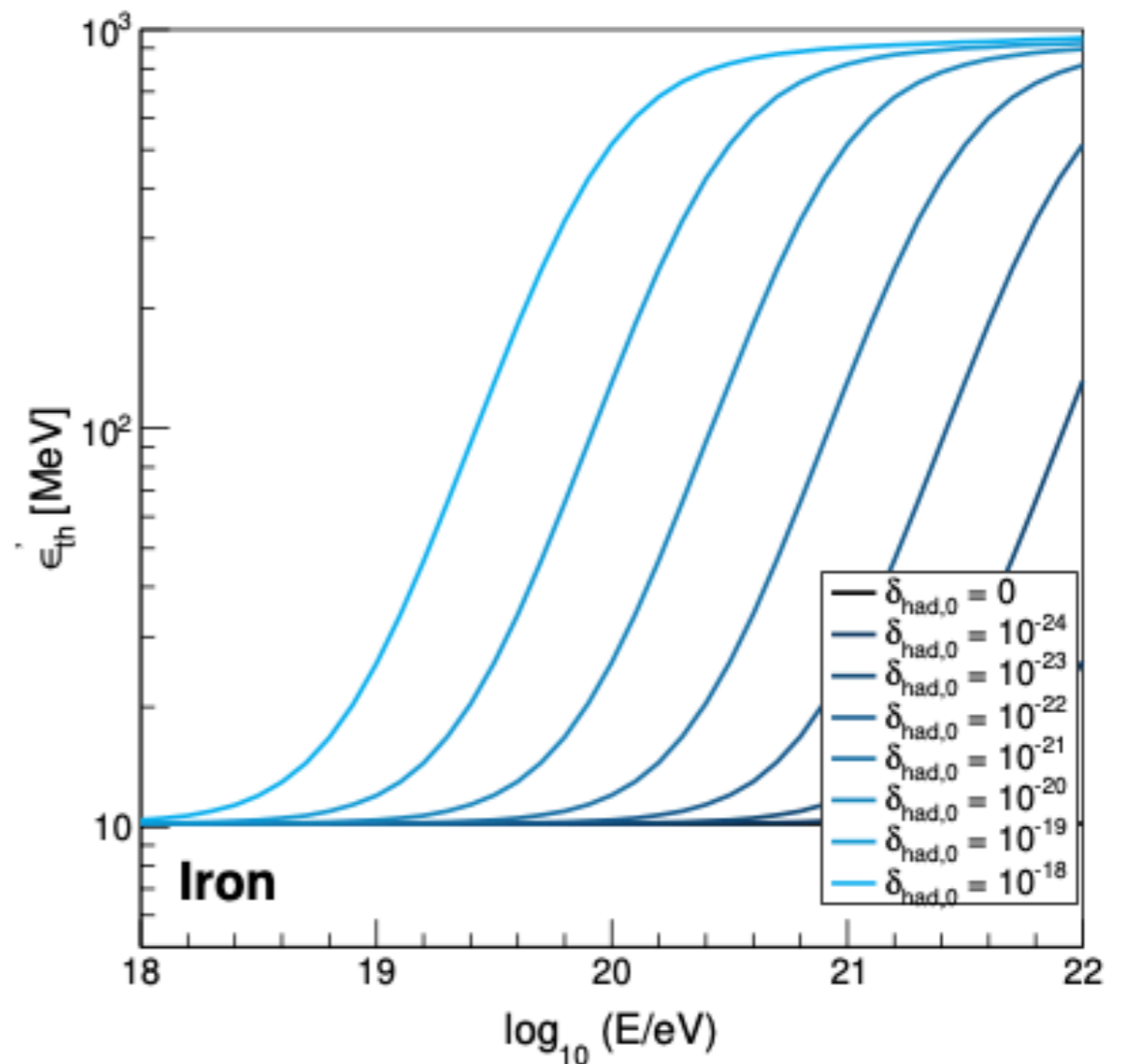
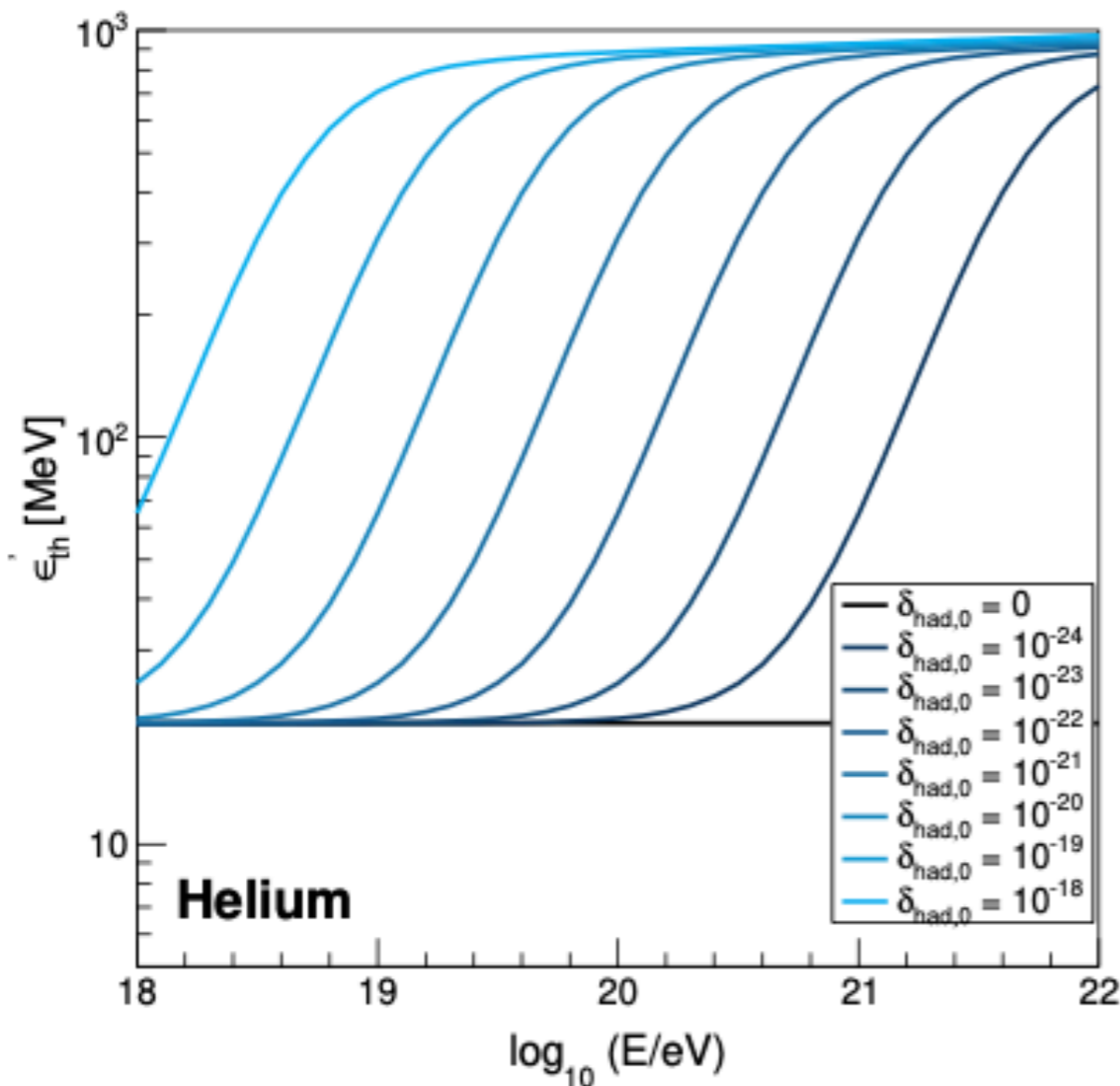
$$\delta_{i,n} = \frac{\eta_{i,n}}{M_{\text{Pl}}^n}$$

is used.



Pierre Auger collaboration, JCAP 01 (2022) 023

**Figure 4.** Attenuation length for photopion production as a function of energy for different LIV coefficients. The black line represents LI  $\delta_{\text{had},0} = 0$  and lines in shades of blue represent different LIV coefficients. Strong LIV with  $\delta_{\text{had}} > 10^{-21}$  results in attenuation lengths larger than  $10^5$  Mpc for all the considered energies and, thus, are not shown in the plot.

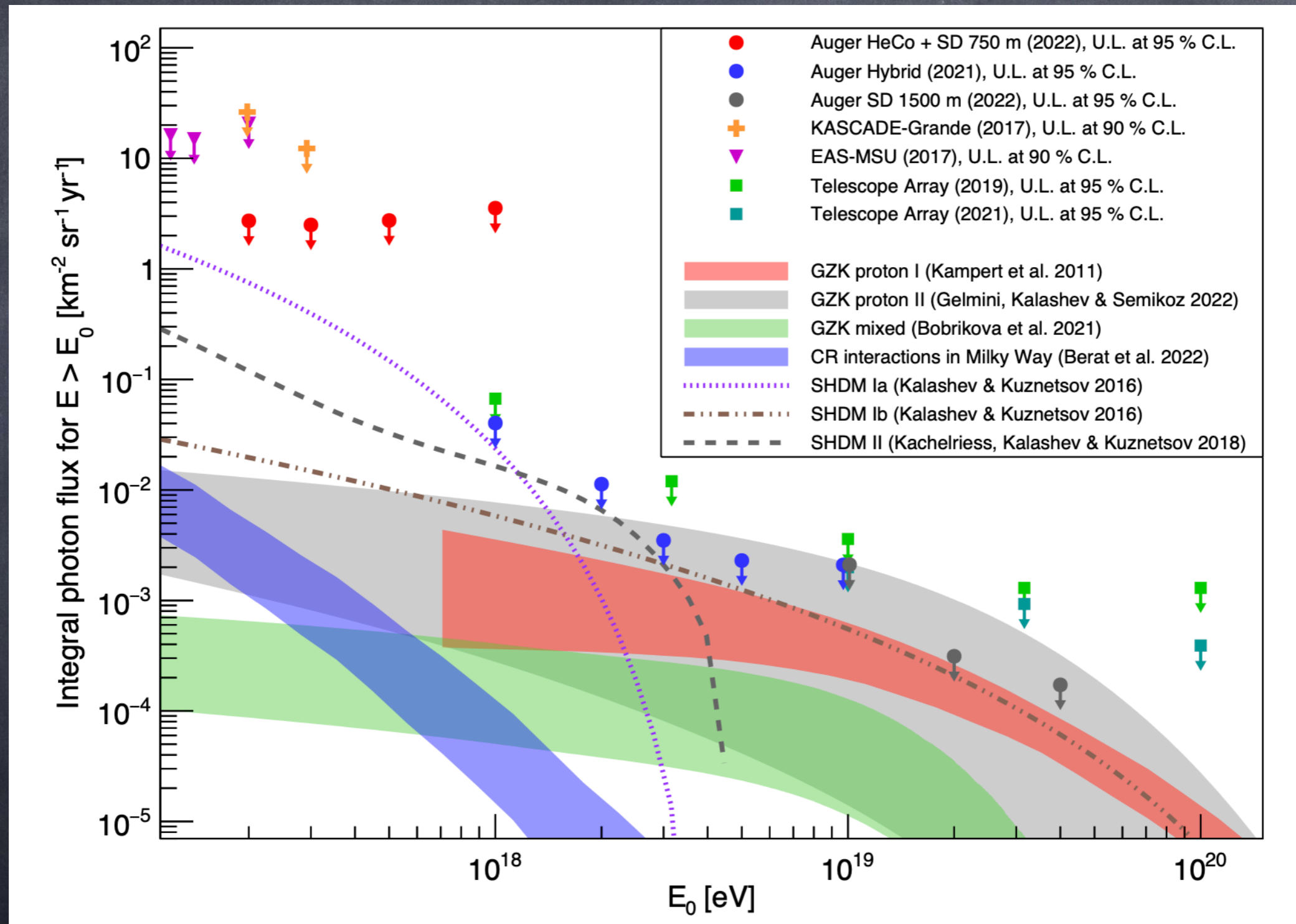


Pierre Auger collaboration, JCAP 01 (2022) 023

**Figure 5.** Energy threshold in the nucleus reference frame for photodisintegration as a function of energy for different LIV coefficients. The black lines represent the LI scenario while the shades of blue represent different LIV coefficients. The left and right panels show the results for a nucleus of helium and iron, respectively.

# Lorentz Symmetry Violation in the Electromagnetic Sector

## Experimental upper limits on UHE photon fluxes

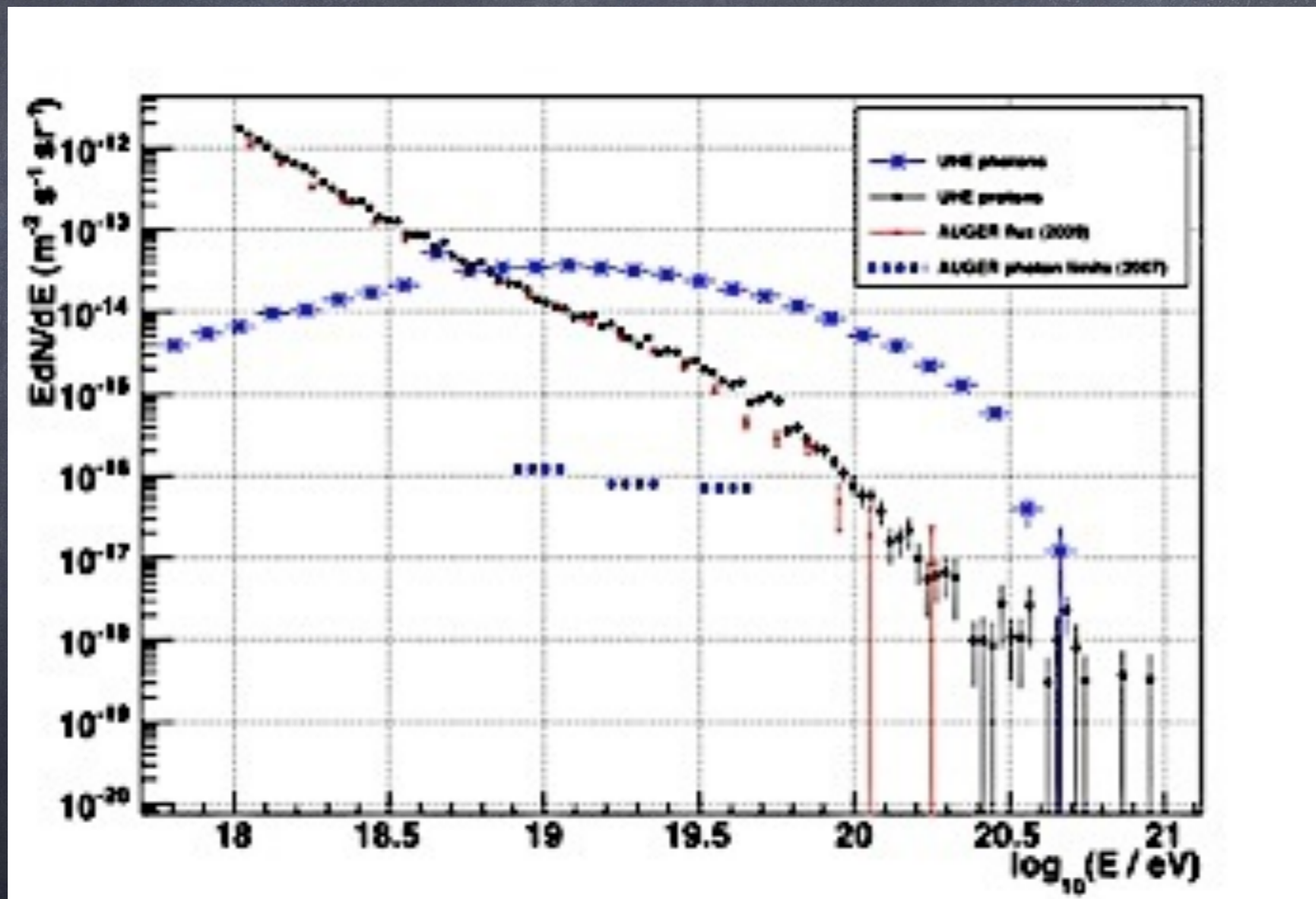


Pierre Auger Collaboration, arXiv:2210.12959, to appear in Universe

# Lorentz Symmetry Violation in the Electromagnetic Sector

The idea:

Photon flux predictions are much higher than experimental upper limits if pair production is suppressed by LIV



Maccione, Liberati, Sigl, PRL 105 (2010) 021101



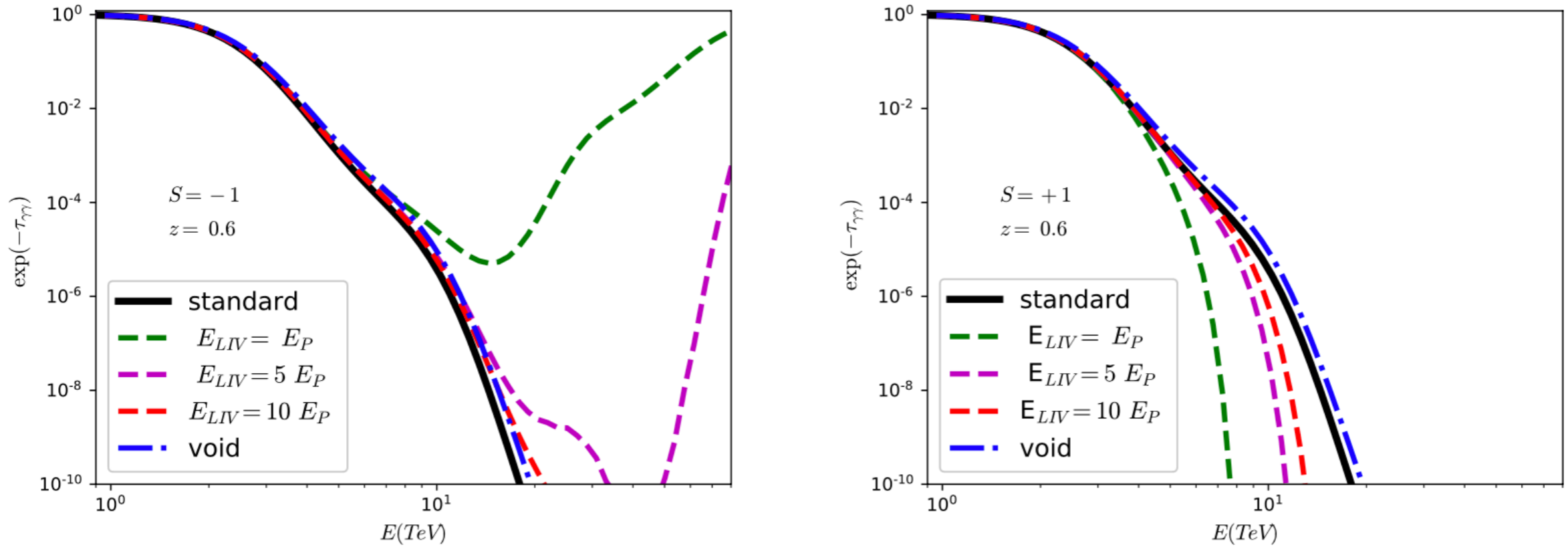


FIG. 1: Gamma-ray absorption coefficients as a function of gamma-ray energy for a source at redshift  $z_s = 0.6$ . The subluminal (here marked by  $S = -1$ ) and superluminal (here marked by  $S = +1$ ) scenarios are represented in the left and right plots, respectively. The standard special-relativistic scenario is represented by solid black lines in both plots, while dashed lines represent modified absorption for different values of quantum gravity energy scale. In all cases  $n = 1$  modification was considered. The dot-dashed blue line represents another effect investigated by the authors not connected to LIV. Figure adopted from [39]. Reproduced by permission of the AAS.

Abdalla and Böttcher, *ApJ* 865 (2018) 159

# Lorentz Symmetry Violation in the Photon Sector

For photons we assume the dispersion relation

$$\omega_{\pm}^2 = k^2 + \xi_n^{\pm} k^2 \left( \frac{k}{M_{\text{Pl}}} \right)^n, n \geq 1,$$

and for electrons

$$E_{e,\pm}^2 = p_e^2 + m_e^2 + \eta_n^{e,\pm} p_e^2 \left( \frac{p_e}{M_{\text{Pl}}} \right)^n, n \geq 1,$$

with only one term present. Polarizations denoted with  $\pm$ . For positrons, effective field theory implies  $\eta_n^{p,\pm} = (-1)^n \eta_n^{e,\pm}$ . Furthermore,  $\xi_n^+ = (-1)^n \xi_n^-$ , so that the problem depends on three parameters which in the following we denote by

$$\xi_n, \eta_n^+, \eta_n^-$$

for each  $n$  (CP-odd and CP-even). Note that **positive/negative coefficients correspond to superluminal/subluminal propagation.**

The original work assumed a pure proton primary cosmic ray composition. This has meanwhile turned out to be unlikely, so that predicted photon fluxes are much smaller and constraints are weaker or even absent. An UHE proton component would help !

Consider pair production on a background photon of energy  $k_b$  and assume kinematics with ordinary energy-momentum conservation, with  $p_e = (1-y)k$ ,  $p_p = yk$ . Using  $x = 4y(1-y)k/k_{LI}$  with the threshold in absence of Lorentz invariance (LI) violation,  $k_{LI} = m_e^2/\omega_b$ , the condition for pair production is then

$$\alpha_n x^{n+2} + x - 1 \geq 0$$

where

$$\alpha_n = \frac{\xi_n - (-1)^n \eta_n^\mp y^{n+1} - \eta_n^\pm (1-y)^{n+1}}{2^{2(n+2)} y^{n+1} (1-y)^{n+1}} \frac{m_e^{2(n+1)}}{k_b^{n+2} M_{Pl}^n}.$$

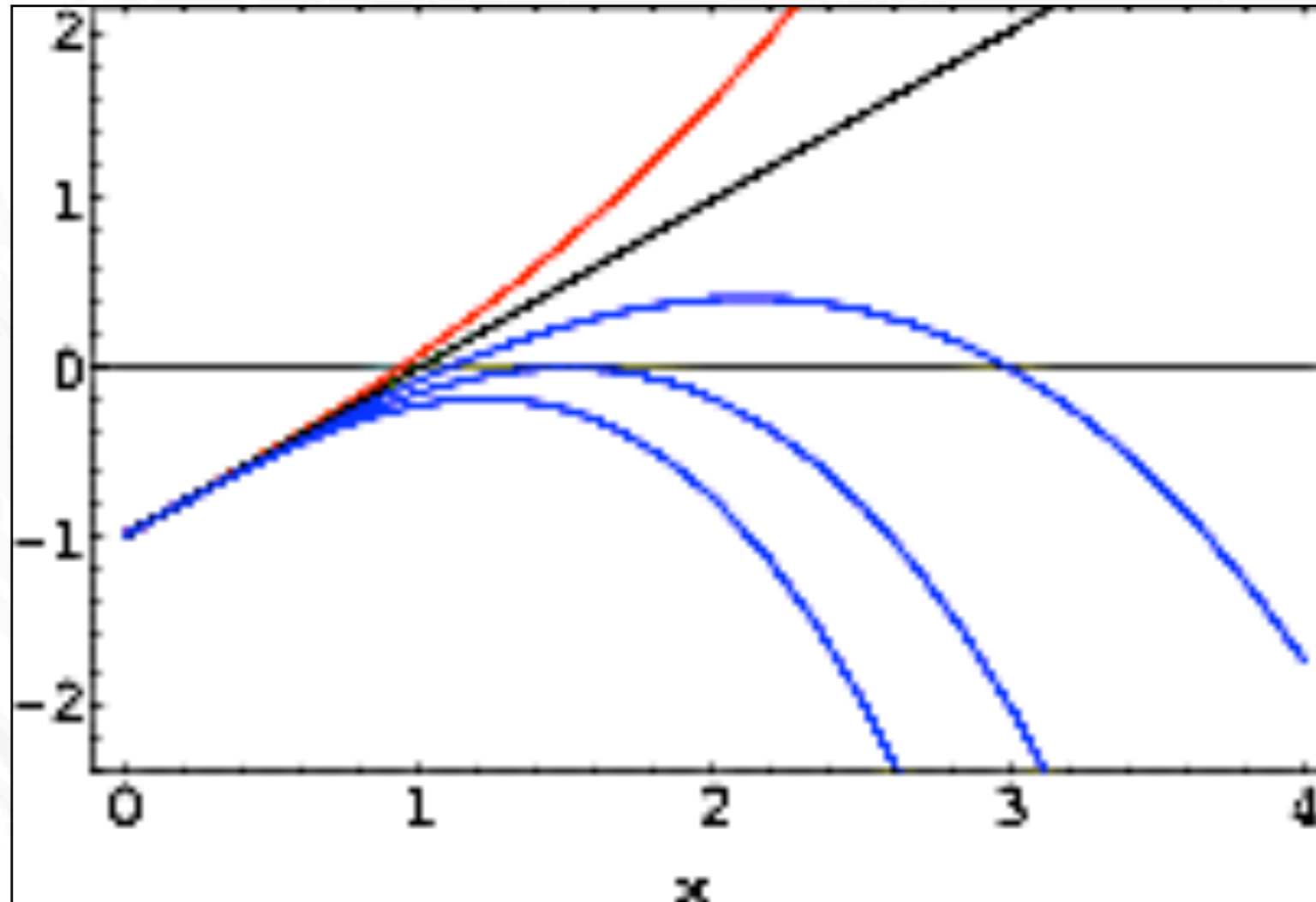
All combinations of  $\xi_n, \eta_n^+, \eta_n^-$  can occur, depending on the partial wave of the pair, governed by total angular momentum conservation. All partial waves are allowed away from the thresholds.

The condition for photon decay is

$$\alpha_n x^{n+2} - 1 \geq 0$$

Rule of thumb: Positive photon LIV coefficient leads to decay and suppressed flux, negative coefficient leads to suppressed pair production and increased flux

There are at most two real solutions  $0 \leq x_n^l \leq x_n^r$  for pair production  
(lower and upper thresholds):



Galaverni, Sigl, Phys. Rev. Lett. 100 (2008) 021102.

For photon decay there is at most one positive real threshold, for superluminal propagation.

Minimize/maximize thresholds with respect to  $y$ .

A given combination  $\xi_n, \eta_n^+, \eta_n^-$  is ruled out if, for  $10^{19} \text{ eV} < \omega < 10^{20} \text{ eV}$ , at least one photon polarization state is stable against decay and does not pair produce for any helicity configuration of the final pair.

In the absence of LIV in pairs,  $\eta_n^+ = \eta_n^- = 0$ , for  $n=1$ , this yields:

$$|\xi_1| \lesssim 10^{-14}$$

and for  $n=2$ :

$$\xi_2 \gtrsim -10^{-6}$$

If a UHE photon were detected, any LIV parameter combination for which photons of both polarisations can decay into at least one helicity configuration of the final pair would be ruled out.

**For  $n = 1$ , all parameters of absolute value  $> 10^{-14}$  ruled out**

**For  $n = 2$ , if absolute value of both the photon and one of the electron parameters is  $< 10^{-6}$ , the second electron parameter can be arbitrarily large even once a UHE photon is seen.**

**Such strong limits may indicate that Lorentz invariance violations are completely absent !**

## Constraints for $n=1$

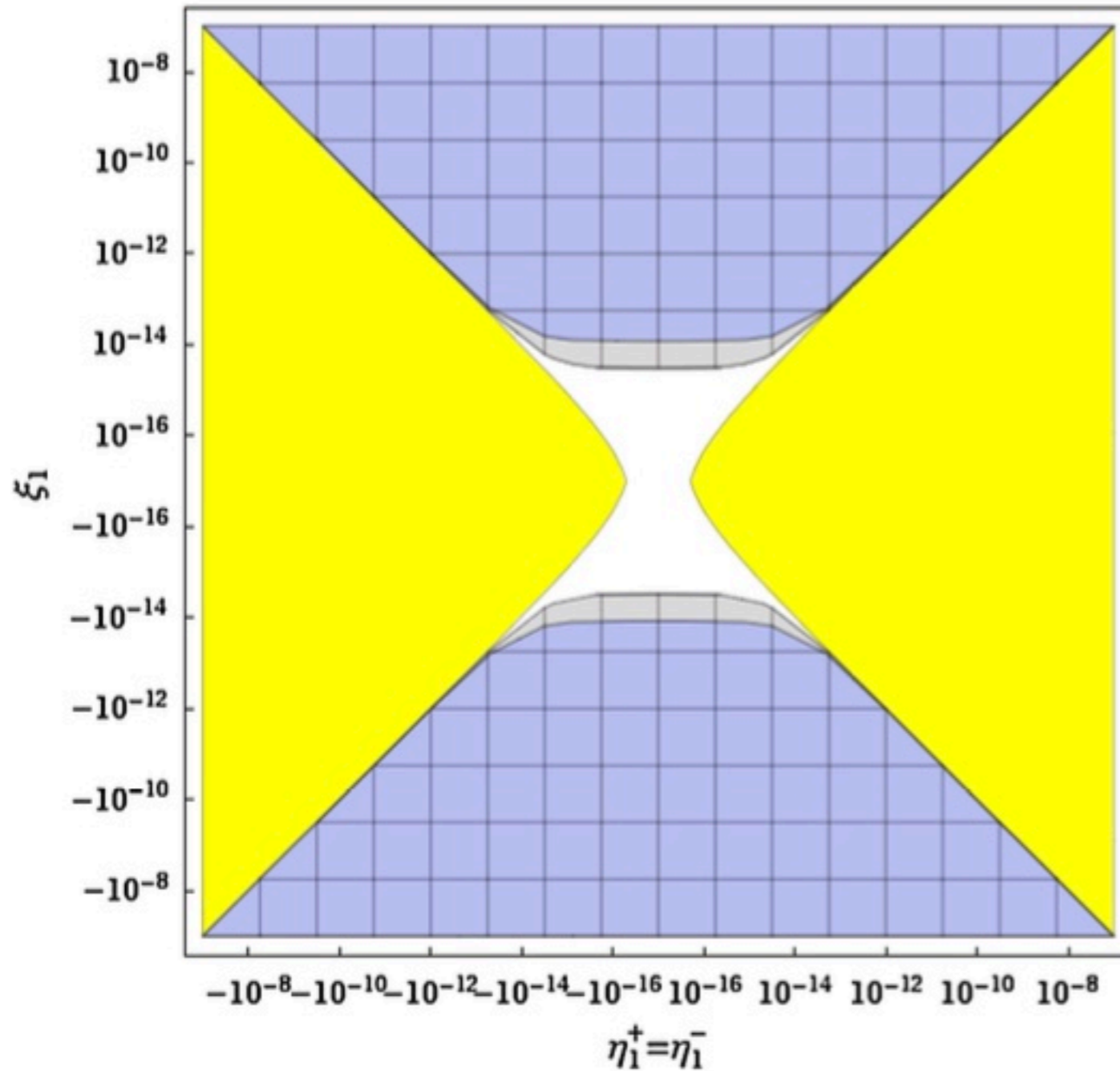
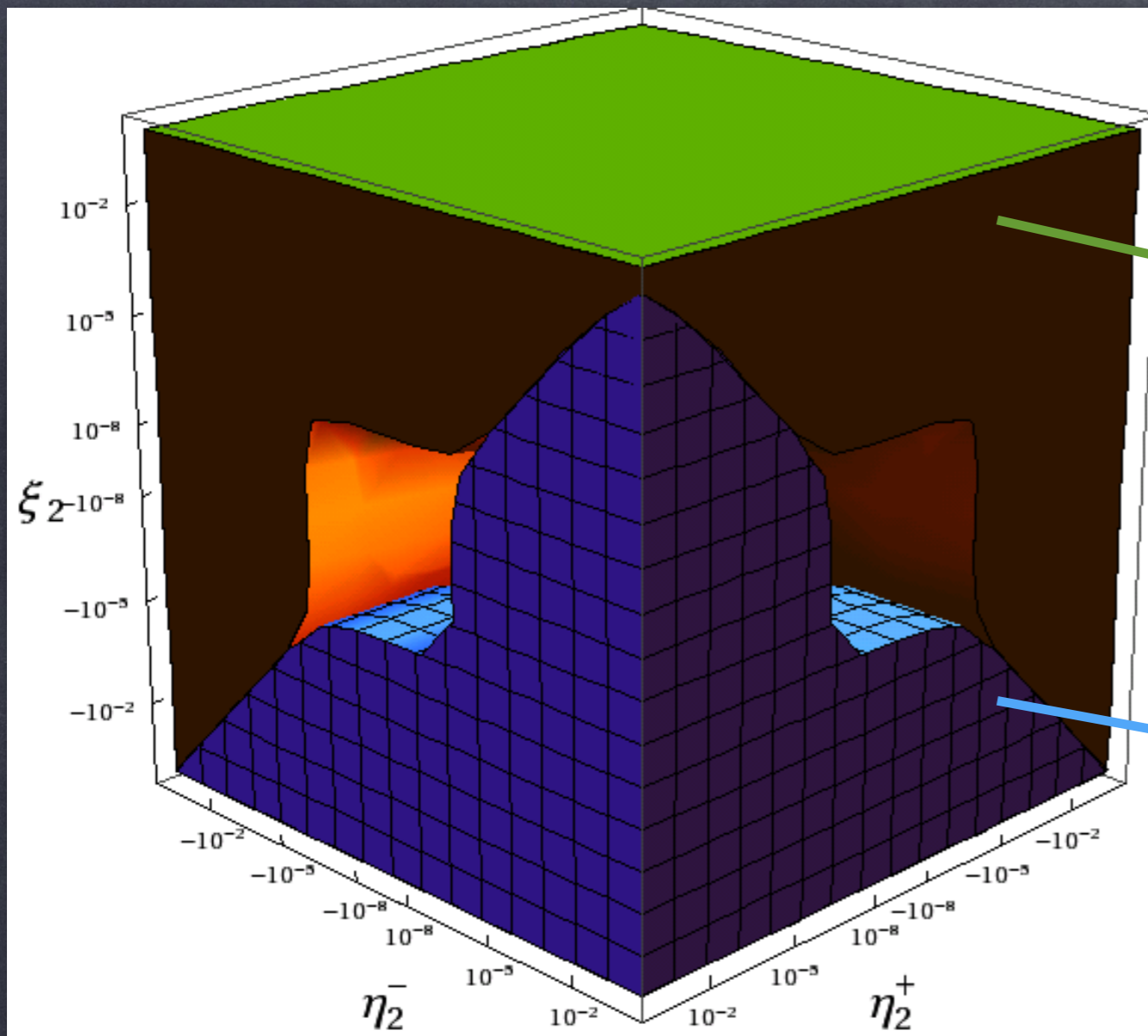


FIG. 4 (color online). Case  $n = 1$ ,  $\eta_1^+ = \eta_1^-$ . Combined constraint using the current upper limits on the photon fraction in the energy range between  $10^{19}$  and  $10^{20}$  eV (gray plus blue shaded, checkered regions), in the energy range between  $10^{19}$  and  $5 \times 10^{19}$  eV (blue region), and assuming that a  $10^{19}$  eV photon were detected (yellow shaded region).

Galaverni, Sigl, PRD 78 (2008) 063003



Constraints for  $n=2$

excluded if UHE photons were detected  
with  $10^{19} \text{ eV} \lesssim E_\gamma \lesssim 10^{20} \text{ eV}$

excluded by current UHE photon  
flux upper limit

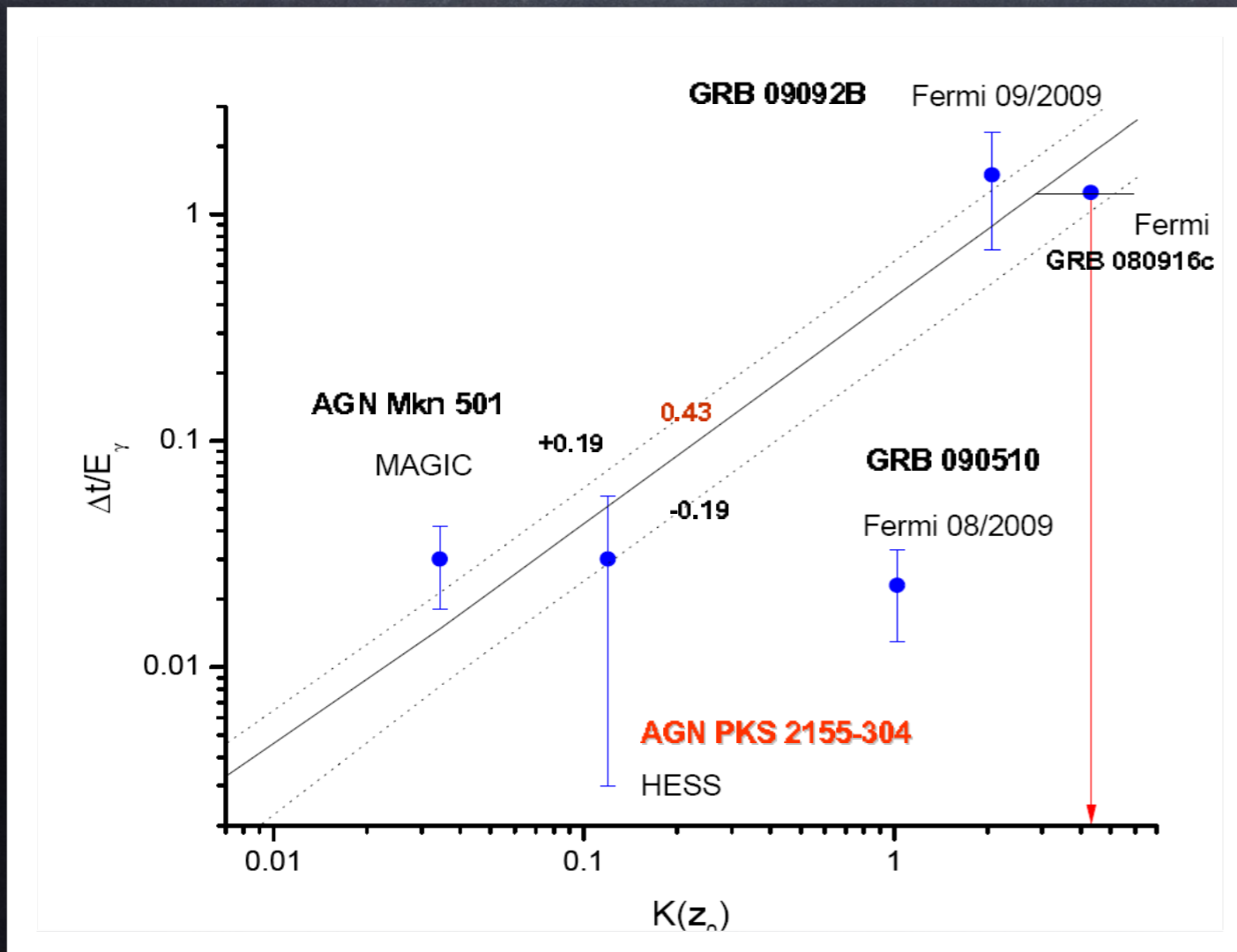
Galaverni, Sigl, PRD 78 (2008) 063003

Such strong limits suggest that Lorentz  
invariance violations are completely absent !

The modified dispersion relation also leads to energy dependent group velocity  $V = \partial E / \partial p$  and thus to an energy-dependent time delay over a distance  $d$ :

$$\Delta t = -\xi d \frac{E}{M_{\text{Pl}}} \simeq -\xi \left( \frac{d}{100 \text{ Mpc}} \right) \left( \frac{E}{\text{TeV}} \right) \text{ sec}$$

for linearly suppressed terms. GRB observations in TeV  $\gamma$ -rays can therefore probe quantum gravity and may explain that higher energy photons tend to arrive later (Ellis, Mavromatos et al.).



Ellis, Mavromatos, arXiv:1111.1178

sensitivity to  $\xi \gtrsim 10$ , close to Planck scale but up to now no clear signature, thus upper limits are generally put.

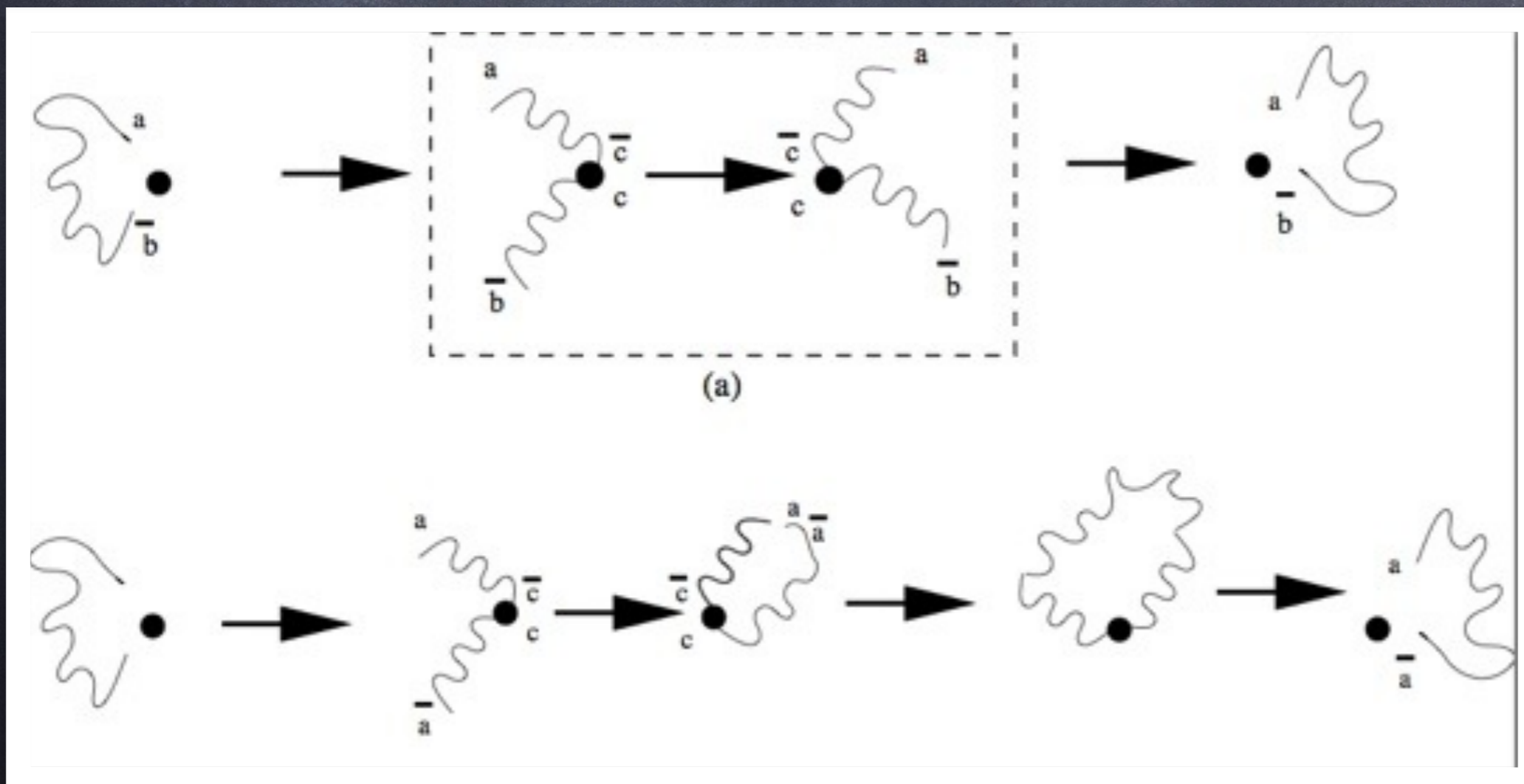


But the UHE photon limits are inconsistent with interpretations of time delays of high energy gamma-rays from GRBs within quantum gravity scenarios based on effective field theory

Maccione, Liberati, Sigl, PRL 105 (2010) 021101

Possible exception in space-time foam models through "noise term" in energy momentum conservation

Ellis, Mavromatos, Nanopoulos, Phys.Lett. B 694 (2010) 61 [arXiv:1004.4167]

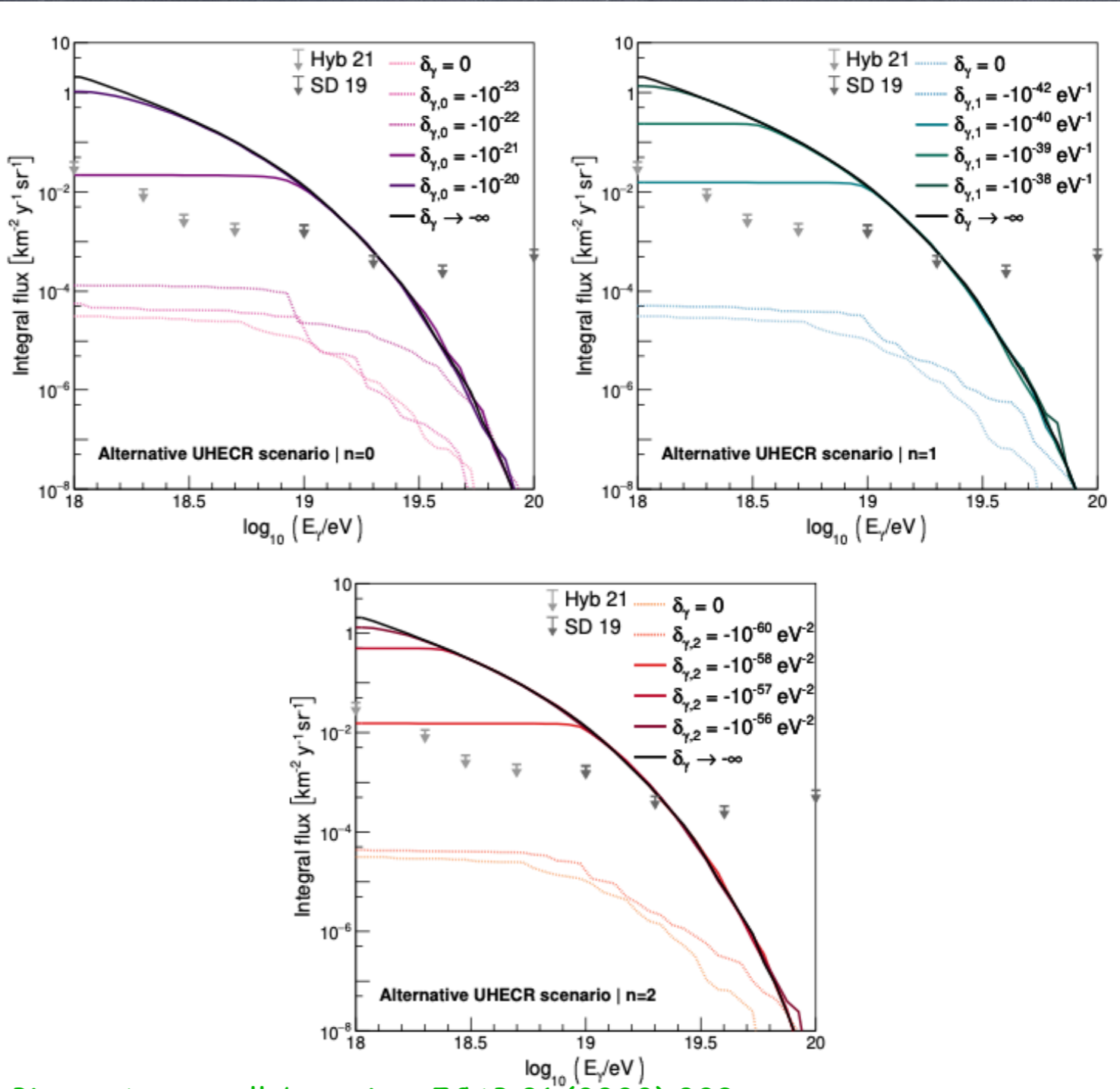


# Updated Photon Constraints based on Pierre Auger data

Here the notation

$$\delta_{i,n} = \frac{\eta_{i,n}}{M_{Pl}^n}$$

is used.



Pierre Auger collaboration, JCAP 01 (2022) 023

**Figure 3.** Simulated integral flux of GZK photons as a function of the energy for an alternative scenario with subdominant proton component [40]. Continuous lines show the rejected LIV scenarios. The arrows show the flux determined by analysis of the Pierre Auger Observatory data [19, 20].

Neglecting LIV in the electron sector, the updated constraints are

$$n = 0 : \delta_{\gamma,0} = \eta_{\gamma,0} \gtrsim -10^{-21}$$

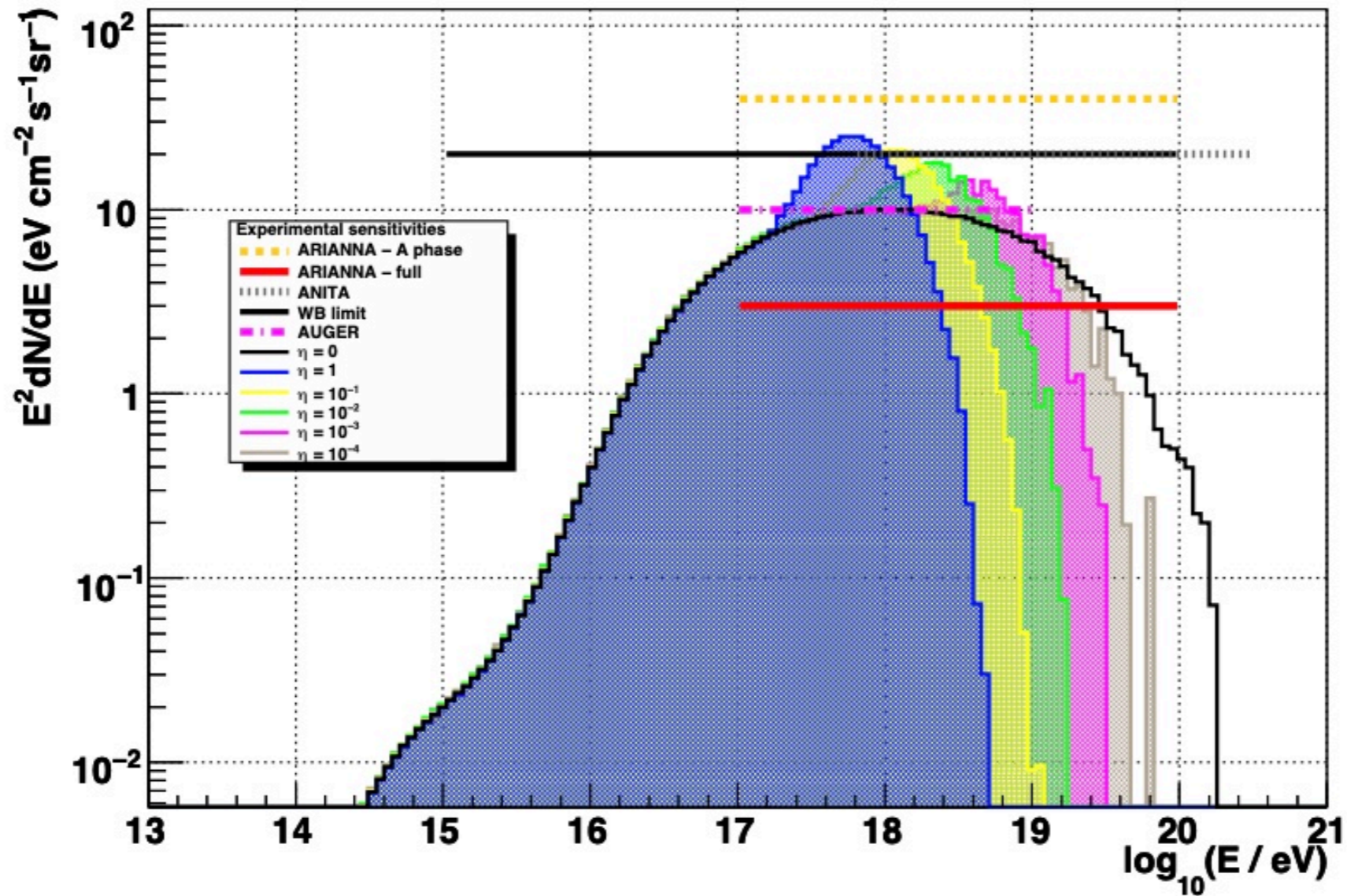
$$n = 1 : \delta_{\gamma,1} \gtrsim -10^{-40} \text{eV}^{-1}, \quad \eta_{\gamma,1} \gtrsim -10^{-12}$$

if relation between different photon polarisations in EFT is taken into account, this turns into an upper bound on absolute value:

$$n = 1 : |\delta_{\gamma,1}| \lesssim 10^{-40} \text{eV}^{-1}, \quad |\eta_{\gamma,1}| \lesssim 10^{-12}$$

$$\text{for } n = 2 : \delta_{\gamma,2} \gtrsim -10^{-58} \text{eV}^{-2}, \quad \eta_{\gamma,2} \gtrsim -0.015$$

Similarly to cosmogenic photons, modified neutrino dispersion relation can lead to neutrino decay. Therefore, observation of a neutrino implies non-decay and thus constraints on LIV parameter



**Figure 2.** Evolution of the predicted LV neutrino spectra varying  $\eta_\nu$  in the “best case scenario”. Sensitivities of main UHE neutrino operating and planned experiments are shown, as found in [47, 51, 67]. The Waxman & Bahcall limit [68, 69] in the interesting energy range is shown for reference.

# Lorentz Symmetry Violation Effects on Air Showers

Main idea is that modified decay rates of neutral and/or charged pions and muons can change shower characteristics such as the muon content and  $X_{\max}$

This could also induce threshold effects e.g. in the muon content as function of primary energy

For example, for QED by a term

$$-\frac{1}{4}(k_F)_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

with  $(k_F)_{\mu\nu\rho\sigma} \propto \kappa$  the photon phase velocity is related to maximal fermion velocity by

$$v_\gamma = \left( \frac{1 - \kappa}{1 + \kappa} \right)^{1/2} v_{f,\max}$$

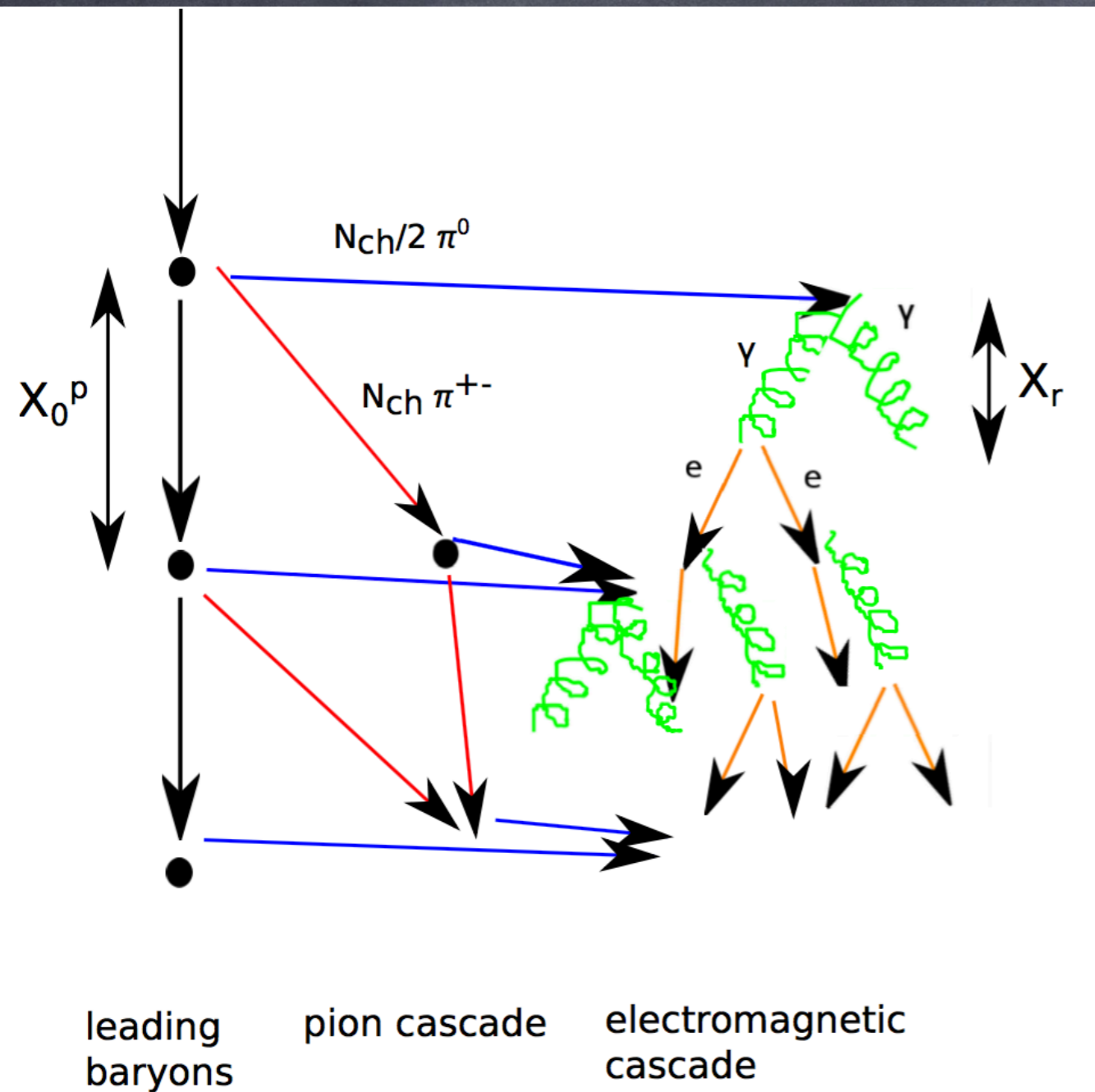
Consider  $\kappa < 0$  which leads to photon decay at energies

$$E_\gamma \gtrsim E_\gamma^{\text{th}} = 2m_e \left( \frac{1 - \kappa}{-2\kappa} \right)^{1/2}$$

and stability of  $\pi^0$  at energies

$$E_{\pi^0} \gtrsim m_{\pi^0} \left( \frac{1 - \kappa}{-2\kappa} \right)^{1/2} \simeq 132 E_\gamma^{\text{th}}$$

# Some Simplified Air Shower Physics



**Fig. 5.2** A sketch of the first two generations of an hadronic cascade in the Heitler Matthews model [232] (left part) and of the first few generations of the electromagnetic cascade in the Heitler model [229] (right part). After each hadronic interaction length  $X_0^p(E)$  the *leading baryon* produces  $N_{ch}(E)$  charged pions and  $N_{ch}(E)/2$  neutral pions. Neutral pions decay into two  $\gamma$ -rays instantaneously whereas charged pions interact again after column depth  $\simeq X_0^p(E)$ , producing further pions. High energy  $\gamma$ -rays produce electron-positron pairs after one radiation length  $X_r$  which in turn recreate  $\gamma$ -rays by bremsstrahlung after a similar length scale.

In this simple picture for a primary energy  $E_p$  the depth of shower maximum is the depth of first interaction  $X_0$  plus the radiation length  $X_r$  times the number of generations  $n$ ,

$$X_{\max} \sim X_0 + X_r \log (E_p/E_c)$$

where  $E_c$  is some critical energy

Muon number increases because stable neutral pions reinteract

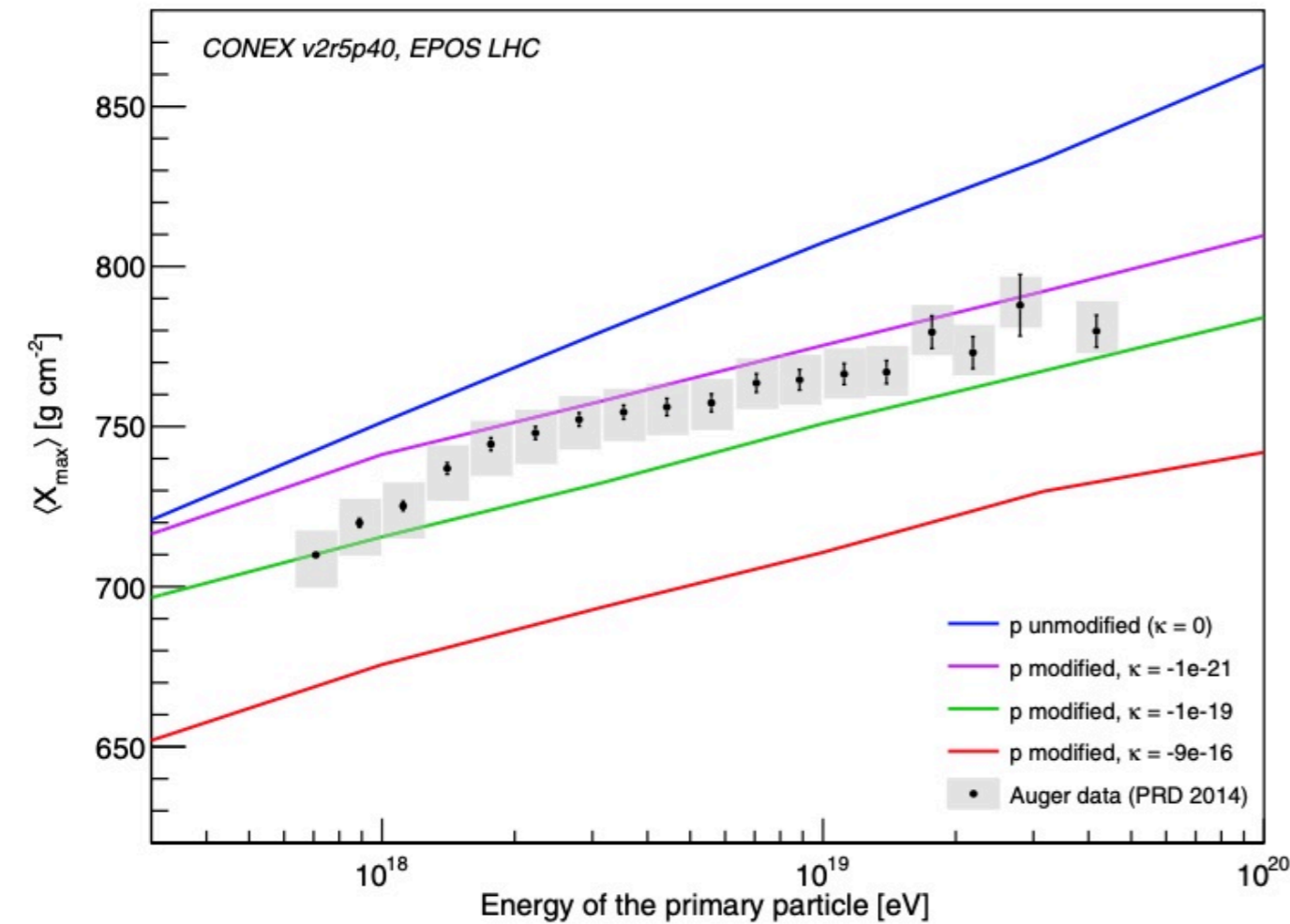


FIG. 5. Simulated values of  $\langle X_{\max} \rangle$  as a function of the primary energy for primary protons compared to measured values of  $\langle X_{\max} \rangle$  by the Pierre Auger Observatory [18]. The gray boxes around the data points indicate the systematic uncertainties of the measurements.

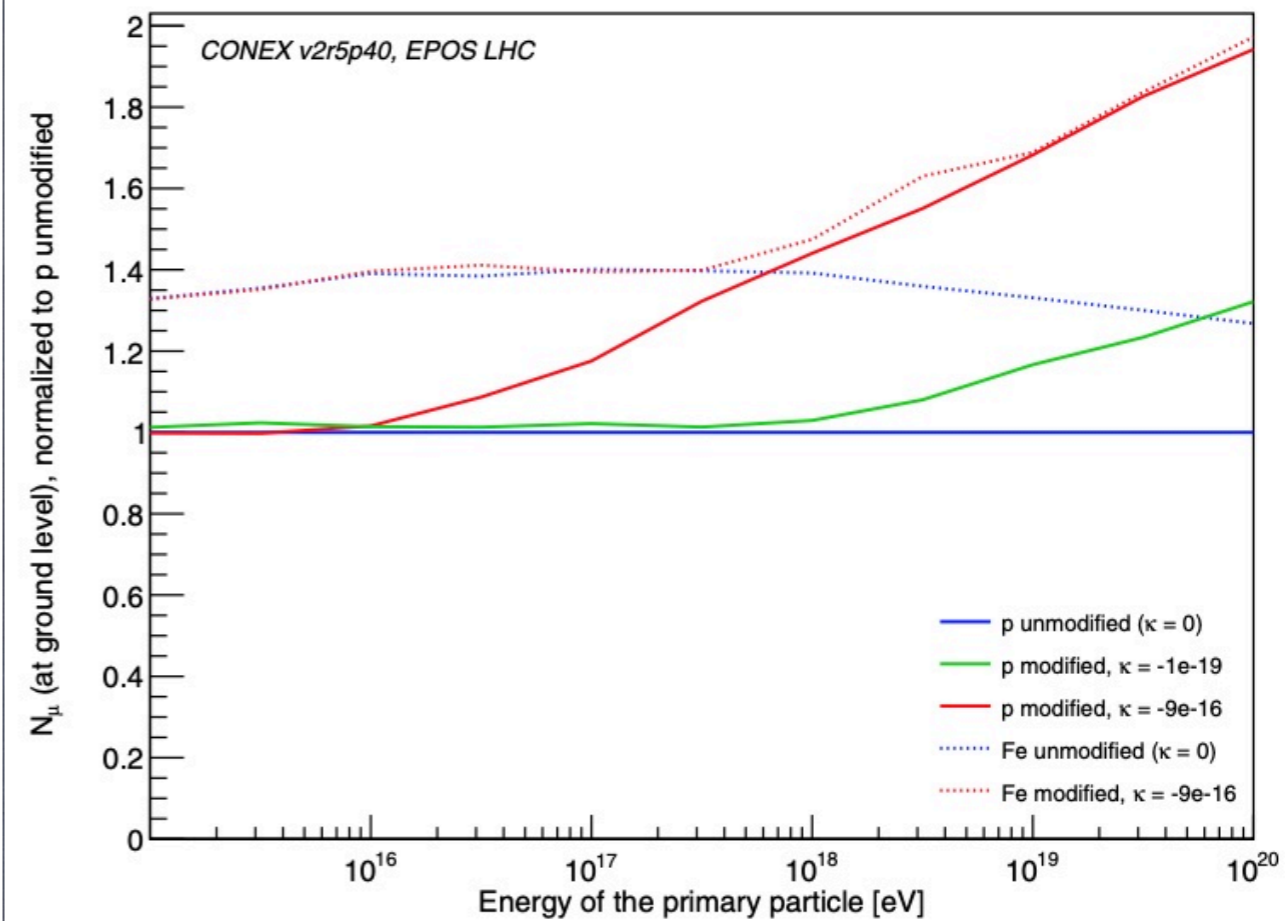


FIG. 7. Average number of ground muons, normalized to the case of unmodified proton primaries, as a function of the primary energy for primary protons and iron nuclei.

Comparison with Pierre Auger  $X_{\max}$  data (photon decay accelerates shower development) then implies the limit

$$\kappa \gtrsim -3 \times 10^{-19}$$



Inclusion of the fluctuations of  $X_{\max}$  data strengthens the lower bound:

$$\kappa \gtrsim -6 \times 10^{-21}$$

Duenkel, Niechciol, Risse, Phys.Rev. D 104 (2021) 015010

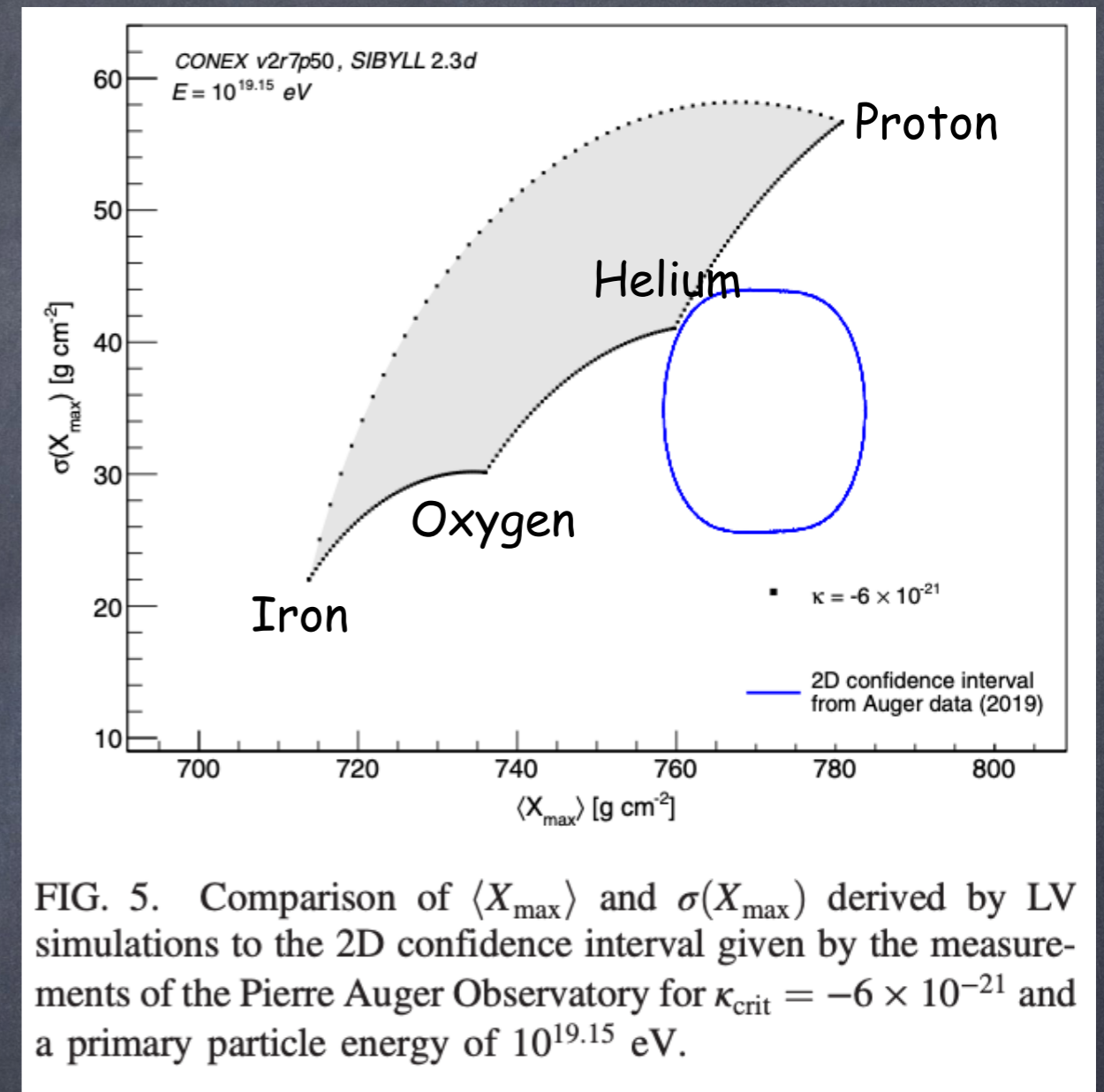


FIG. 5. Comparison of  $\langle X_{\max} \rangle$  and  $\sigma(X_{\max})$  derived by LV simulations to the 2D confidence interval given by the measurements of the Pierre Auger Observatory for  $\kappa_{\text{crit}} = -6 \times 10^{-21}$  and a primary particle energy of  $10^{19.15}$  eV.

Positive  $\kappa$  leads to vacuum Cherenkov radiation  $p \rightarrow p + \gamma$  above a critical energy. Based on the highest energy event seen by Pierre Auger this gives

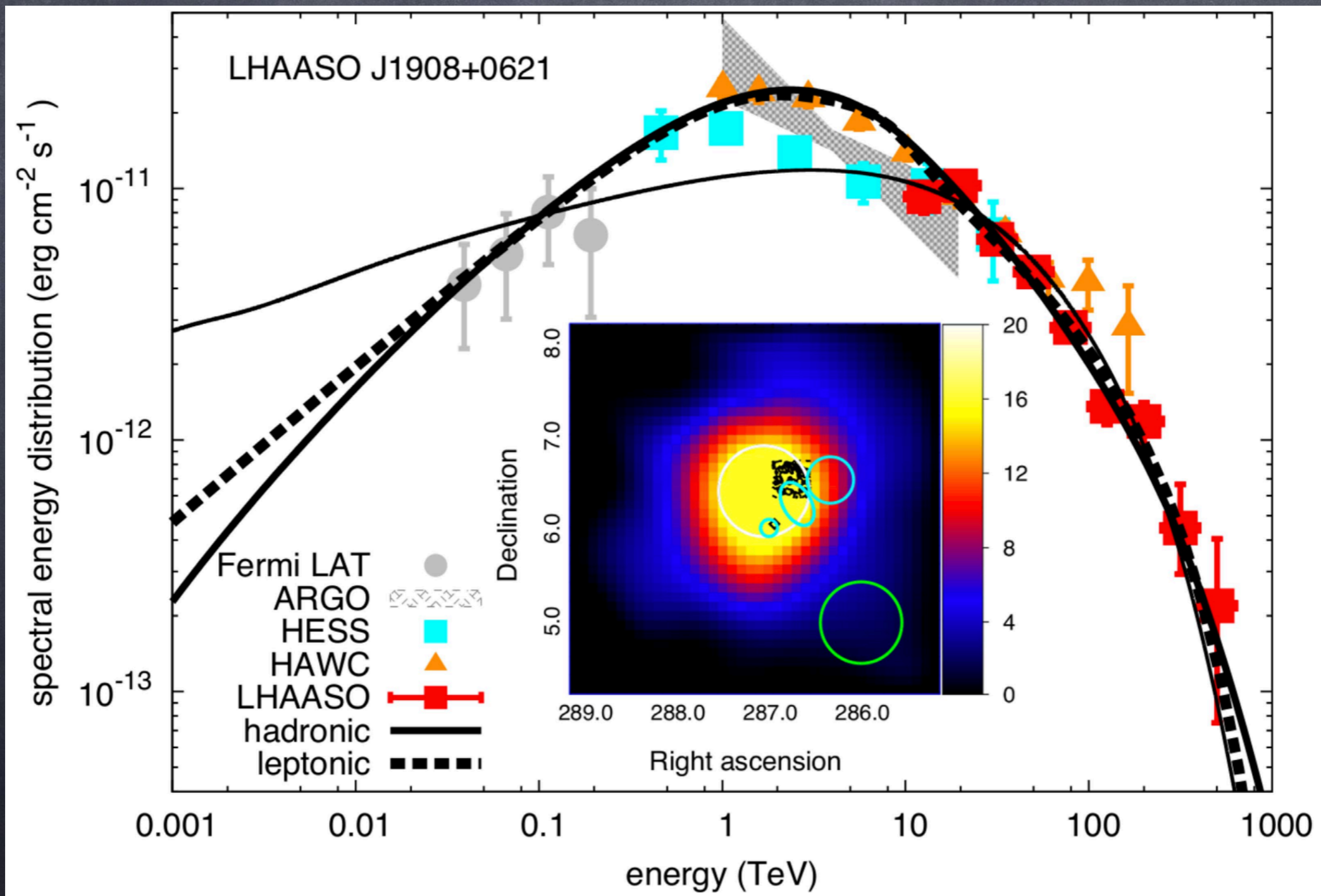
$$\kappa \lesssim 6 \times 10^{-20}$$

Klinkhamer, Schreck, Phys.Rev. D 78 (2008) 085026

# Electromagnetic Lorentz Symmetry Violation Constraints from LHAASO

Photons from galactic sources (pulsars) have been observed up to PeV energies

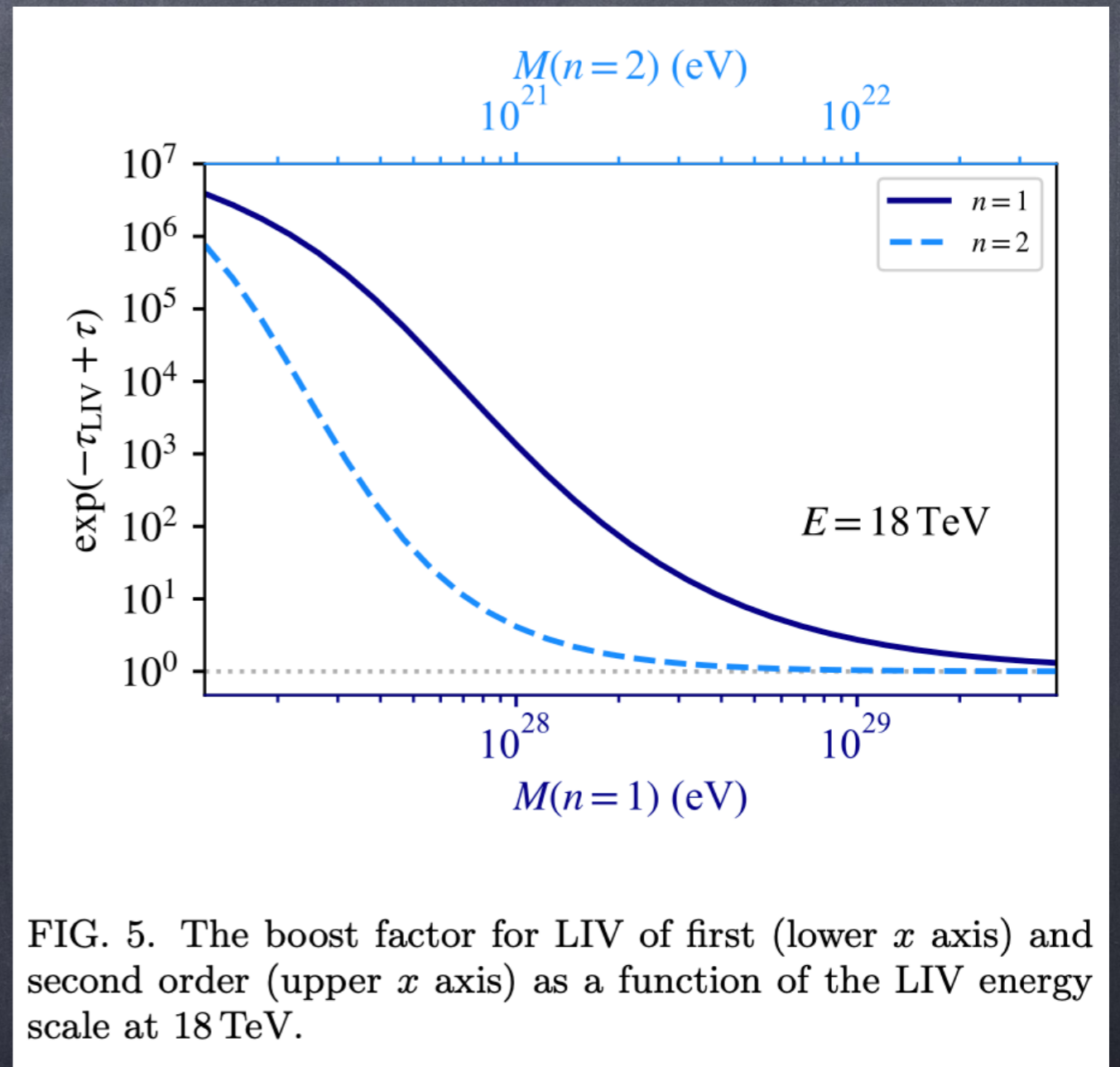
Cao et al., LHAASO collaboration, Nature 594 (2021) 33



The observation of a  $\sim 18$  TeV photon from the GRB source GRB221009A from a redshift  $z=0.1505$  requires a boost factor of  $\sim 10^6$  beyond the predicted gamma-ray attenuation by pair production on the infrared background. In the context of Lorentz invariance violation this could be explained by

$$\eta_{\gamma,1} \lesssim -1 \text{ or } \eta_{\gamma,2} \lesssim -10^{-9}.$$

But note that this contradicts the Pierre Auger limits (it would probably imply that Auger should see a large gamma-ray flux)



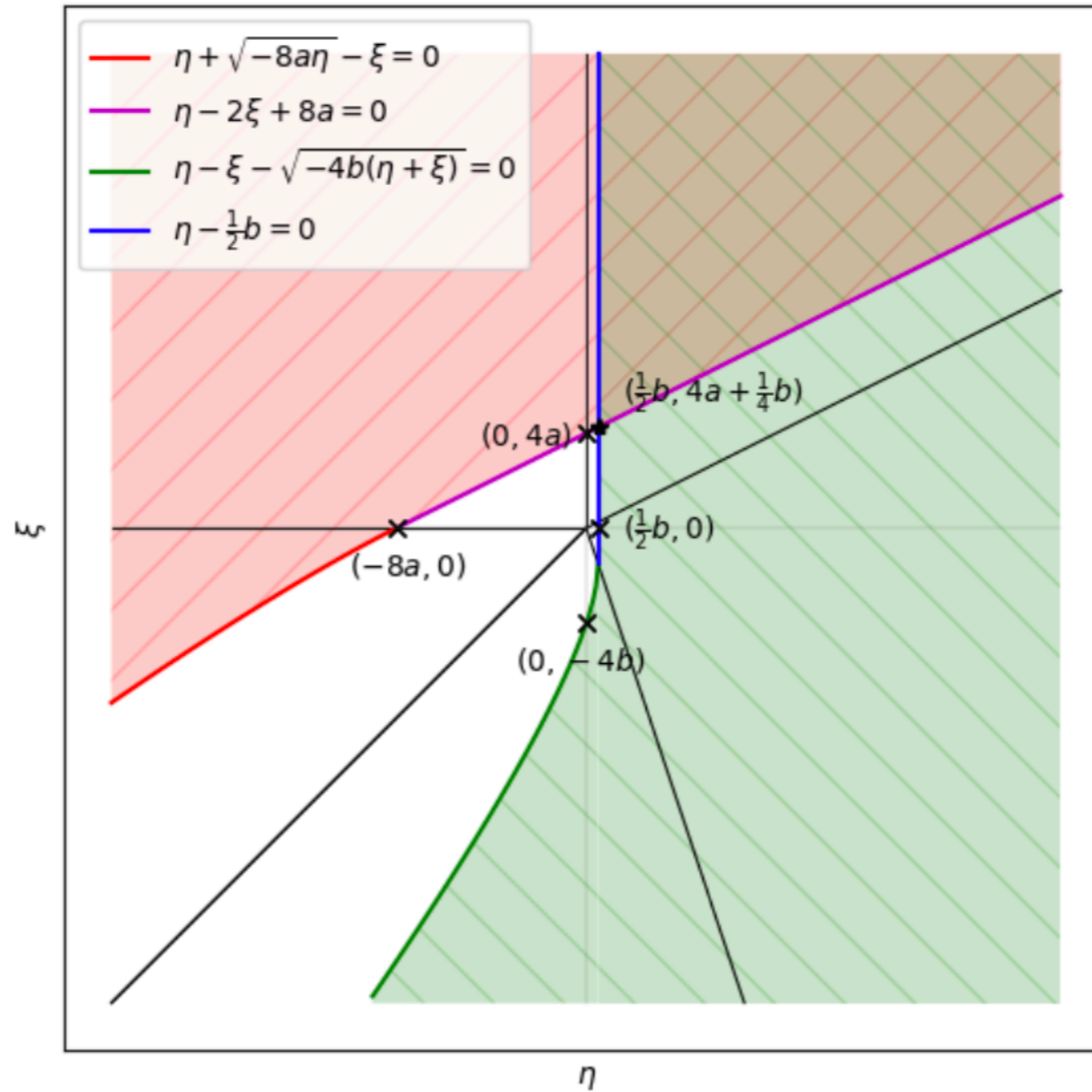


Figure 3: Joint constraint on photon-electron LV parameter plane from highest-energy photon and electron ( $a \equiv m^2 E_{P1}/E_p^3$  and  $b \equiv m^2 E_{P1}/E_e^3$ ).

# Conclusions

- 1.) Both cosmic ray propagation and air showers can constrain Lorentz symmetry violations, sometimes providing the strongest possible constraints due to the high energies available.
- 2.) More work is needed, for example on the update of photon- and neutrino-based constraints for a mixed composition.
- 3.) Establishing/distinguishing existence of interactions (such as GZK) versus collision-less source physics would strengthen sensitivity to LIV (more statistics needed)
- 4.) Constraints are often limited to specific scenarios -> generalisation of underlying theoretical scenarios desirable.