

$$\begin{cases} \frac{dx}{dt} = v_x \\ \frac{dy}{dt} = v_y \\ \frac{dv_x}{dt} = -Gm\frac{x}{r^3} \\ \frac{dv_y}{dt} = -Gm\frac{y}{r^3} \end{cases}$$

$$r = \sqrt{x^2 + y^2}$$

$$\frac{d}{dt} \begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix} = f \begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix} = \begin{pmatrix} 0 & 0 & v_x & 0 \\ 0 & 0 & 0 & v_y \\ -\frac{Gmx}{r^3} & 0 & 0 & 0 \\ 0 & -\frac{Gmy}{r^3} & 0 & 0 \end{pmatrix}$$

$$u = (x, y, v_x, v_y)$$

$$\frac{\partial f}{\partial u} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ Gm \left( \frac{x^2}{r^3} - \frac{1}{r} \right) & Gm \left( \frac{xy}{r^3} \right) & 0 & 0 \\ Gm \left( \frac{xy}{r^3} \right) & Gm \left( \frac{y^2}{r^3} - \frac{1}{r} \right) & 0 & 0 \end{pmatrix}$$

wartości własne:

$$\sigma = \left\{ 0, 0, -i\sqrt{\frac{Gm}{r}}, i\sqrt{\frac{Gm}{r}} \right\}$$

$$\Delta t \leq \text{const} \sqrt{\frac{r}{Gm}}$$