

$$\begin{cases} \frac{dx}{dt} = +\omega v \\ \frac{dv}{dt} = -\omega x \end{cases}$$

$$\begin{cases} x^{n+1} = x^n + \frac{\omega \Delta t}{2}(v^n + v^{n+1}) \\ v^{n+1} = v^n - \frac{\omega \Delta t}{2}(x^n + x^{n+1}) \end{cases}$$

$$\begin{cases} x^{n+1} - \frac{\omega \Delta t}{2}v^{n+1} = +x^n + \frac{\omega \Delta t}{2}v^n & | \times \frac{\omega \Delta t}{2} - \\ \frac{\omega \Delta t}{2}x^{n+1} + v^{n+1} = -\frac{\omega \Delta t}{2}x^n + v^n & | \times \frac{\omega \Delta t}{2} + \end{cases}$$

$$\begin{cases} x^{n+1} \left(1 + \frac{\omega^2 \Delta t^2}{4}\right) = x^n \left(1 - \frac{\omega^2 \Delta t^2}{4}\right) + v^n \omega \Delta t \\ v^{n+1} \left(1 + \frac{\omega^2 \Delta t^2}{4}\right) = v^n \left(1 - \frac{\omega^2 \Delta t^2}{4}\right) - x^n \omega \Delta t \end{cases}$$