$$\nabla^2 \Phi(x) = -\varrho(x) \quad (1)$$

$$\nabla^2 \Phi_j = -\varrho_j \quad (2)$$

$$\frac{\Phi_{j-1} - 2\Phi_j + \Phi_{j+1}}{\Delta^2} = -\varrho_j \quad (3)$$

$$\Phi_j = \sum_{k=0}^{N-1} \phi_k e^{\frac{i2\pi kj}{N}} \quad (4)$$

$$\sum_{k=0}^{N-1} \phi_k (e^{\frac{-i2\pi k}{N}} - 2 + e^{\frac{i2\pi k}{N}}) e^{\frac{i2\pi kj}{N}} = -\Delta^2 \varrho_j \quad (5)$$

$$\sum_{k=0}^{N-1} \phi_k (2\cos\frac{2\pi k}{N} - 2) e^{\frac{i2\pi kj}{N}} = -\Delta^2 \varrho_j$$

$$| \sum_{j=0}^{N-1} (\cdot) e^{\frac{-i2\pi jl}{N}} \quad (6)$$

$$\sum_{j=0}^{N-1}\sum_{k=0}^{N}\phi_k(2\cos\frac{2\pi k}{N}-2)e^{\frac{i2\pi kj}{N}}e^{\frac{-i2\pi jl}{N}}=$$

$$=-\Delta^2\sum_{j=0}^{N-1}\varrho_j e^{\frac{-i2\pi jl}{N}} \tag{7}$$

$$\sum_{k=0}^{N-1}\phi_k(2\cos\frac{2\pi k}{N}-2)\overbrace{\sum_{j=0}^{N-1}e^{\frac{i2\pi kj}{N}}e^{\frac{-i2\pi jl}{N}}}^{N\delta_{kl}}=$$

$$=-\Delta^2\sum_{j=0}^{N-1}\varrho_j e^{\frac{-i2\pi jl}{N}} \tag{8}$$

$$N\phi_k(2\cos\frac{2\pi k}{N}-2)=-\Delta^2\sum_{j=0}^{N-1}\varrho_j e^{\frac{-i2\pi jk}{N}} \tag{9}$$

$$\phi_k = \frac{1}{N\Delta^2(2 - 2\cos\frac{2\pi k}{N})} \sum_{j=0}^{N-1} \varrho_j e^{\frac{-i2\pi jk}{N}}, k \neq 0 \tag{10}$$

$$\phi_0 = \text{const} \tag{11}$$

$$\Phi_j = \sum_{k=0}^{N-1} \phi_k e^{\frac{i2\pi kj}{N}} \tag{12}$$

$$\rho_k = \sum_{j=0}^{N-1} \varrho_j e^{\frac{-i2\pi jk}{N}} \tag{13}$$

$$\begin{aligned} \Re(\rho_k) &= \Re(\rho_{N-k}) \\ \Im(\rho_k) &= -\Im(\rho_{N-k}) \end{aligned} \tag{14}$$